Sinais e Sistemas
Formulário completo

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2 de outubro de 2014

1 Matemática

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha}, N_2 \geq N_1 \quad (1)$$

2 Domínio do tempo

$$E_x = \sum_{k=-\infty}^{+\infty} |x[k]|^2 \, dt \quad (2)$$

$$P_x = \lim_{N\to+\infty} \frac{1}{N} \int_{k_0}^{k_0+N} |x[k]|^2 \, dt \quad (3)$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \quad (4)$$

$$\begin{aligned} y[n] = x[n] * h[n] \Leftrightarrow \\ y[an] = |a| \, x[at] * h[at] \end{aligned} \quad (5)$$

2.1 Equação à diferenças

Seja p o maior atraso de $y[n]$ e q o maior atraso de $x[n]$ de uma equação a diferenças linear com coeficientes constantes.

$$y_p[n] = \begin{cases} 0 & p > q \\ K\delta[n] & p = q \\ \begin{aligned} K_{q-p}\delta[n-(q-p)]+ \\ K_{q-p-1}\delta[n-(q-p-1)]+ \\ \cdots + \\ K_0\delta[n] \end{aligned} & p < q \end{cases} \quad (6)$$

3 Domínio de Fourier

$$e^{\pm jx} = \cos(x) \pm j\,\mathrm{sen}(x) \quad (7)$$

$$\begin{aligned} \cos(x) &= \frac{e^{+jx} + e^{-jx}}{2} \\ \mathrm{sen}(x) &= \frac{e^{+jx} - e^{-jx}}{2j} \end{aligned} \quad (8)$$

$$\begin{aligned} \cos(a \pm b) &= \cos(a)\cos(b) \mp \mathrm{sen}(a)\,\mathrm{sen}(b) \\ \mathrm{sen}(a \pm b) &= \mathrm{sen}(a)\cos(b) \pm \mathrm{sen}(b)\cos(a) \end{aligned} \quad (9)$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} x[k]e^{-j\omega k} \quad (10)$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{+j\omega n} d\omega \quad (11)$$

$$\begin{aligned} X(e^{j\omega}) &= X_p(e^{j\omega}) + X_i(e^{j\omega}) \\ X_p(e^{j\omega}) &= \frac{1}{2}\left[X(e^{j\omega} + X^*(e^{-j\omega}\right] \\ X_i(e^{j\omega}) &= \frac{1}{2}\left[X(e^{j\omega} - X^*(e^{-j\omega}\right] \\ X_p(e^{j\omega}) &= X_p^*(e^{-j\omega}) \\ X_i(e^{j\omega}) &= -X_i^*(e^{-j\omega}) \end{aligned} \quad (12)$$

$$\mathrm{FT_{DT}}\left\{x[-n]\right\} = X(e^{-j\omega}) \quad (13)$$

$$\mathrm{FT_{DT}}\left\{x^*[n]\right\} = X^*(e^{-j\omega}) \quad (14)$$

$$\mathrm{FT_{DT}}\left\{nx[n]\right\} = j\frac{dX(e^{j\omega})}{d\omega} \quad (15)$$

$$\mathrm{FT_{DT}}\left\{1\right\} = \sum_{k=-\infty}^{+\infty} 2\pi\delta(\omega + 2\pi k) \quad (16)$$

$$\mathrm{FT_{DT}}\left\{u[n]\right\} = \frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega + 2\pi k) \quad (17)$$

$$\mathrm{FT_{DT}}\left\{(n+1)a^n u[n]\right\} = \frac{1}{(1-ae^{-j\omega})^2} \quad (18)$$

$$\mathrm{FT_{DT}}\left\{\begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{c.c.} \end{cases}\right\} = \frac{\mathrm{sen}(\omega(M+1)/2)}{\mathrm{sen}(\omega/2)} \times e^{-j\omega M/2} \quad (19)$$

$$\mathrm{FT_{DT}}\left\{\frac{\mathrm{sen}(\omega_c n)}{\pi n}\right\} = \begin{cases}1, & |\omega| < \omega_c \\ 0, & \omega_c < \omega \leq \pi\end{cases} \tag{20}$$

$$\mathrm{FT_{DT}}\left\{e^{\jmath\omega_0 n}\right\} = \sum_{k=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k) \tag{21}$$

4 Domínio Z

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n} \tag{22}$$

$$x[n] = \frac{1}{2\pi\jmath}\oint_C X(z)z^{n-1}dz \tag{23}$$

$$\mathscr{Z}\left\{z_0^n x[n]\right\} = X(z/z_0) \tag{24}$$

$$\mathscr{Z}\left\{x^*[n]\right\} = X^*(z^*) \tag{25}$$

$$\mathscr{Z}\left\{nx[n]\right\} = -z\frac{d}{dz}X(z), \quad \mathrm{ROC} = R_x \tag{26}$$

$$\mathscr{Z}\left\{a^n u[n]\right\} = \frac{1}{1 - az^{-1}}, \quad |z| > |a| \tag{27}$$

$$\mathscr{Z}\left\{-a^n u(-n-1)\right\} = \frac{1}{1 - az^{-1}}, \quad |z| < |a| \tag{28}$$

$$\mathscr{Z}\left\{\delta[n]\right\} = 1 \tag{29}$$

$$\mathscr{Z}\left\{u[n]\right\} = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{30}$$

$$\mathscr{Z}\left\{-u[-n-1]\right\} = \frac{1}{s}, \quad |z| < 1 \tag{31}$$

$$\mathscr{Z}\left\{na^n u[n]\right\} = \frac{az^{-1}}{(1 - az^{-1})^2}, \quad |z| > |a| \tag{32}$$

$$\mathscr{Z}\left\{-na^n u[-n-1]\right\} = \frac{az^{-1}}{(1 - az^{-1})^2}, \quad |z| < |a| \tag{33}$$

$$\mathscr{Z}\left\{r^n \cos(\omega_0 t)u(t)\right\} = \frac{1 - (r\cos(\omega_0))z^{-1}}{1 - (2r\cos(\omega_0))z^{-1} + r^2 z^{-2}}, \quad |z| > |r| \tag{34}$$

$$\mathscr{Z}\left\{r^n \,\mathrm{sen}(\omega_0 t)u(t)\right\} = \frac{(r\cos(\omega_0))z^{-1}}{1 - (2r\cos(\omega_0))z^{-1} + r^2 z^{-2}}, \quad |z| > |r| \tag{35}$$

$$\mathrm{Res}\left[X(z)z^{n-1} \text{ em } z = z_0\right] = \frac{1}{(s-1)!}\left[\frac{d^{s-1}\Psi(z)}{dz^{s-1}}\right]_{z=z_0}, \quad \Psi(z) = \left[X(z)z^{n-1}\right](z - z_0)^s \tag{36}$$