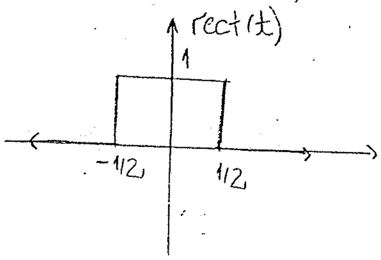
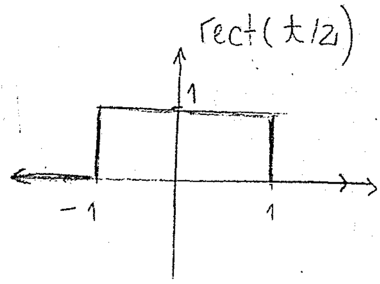
Lista - Sinais e sistemas

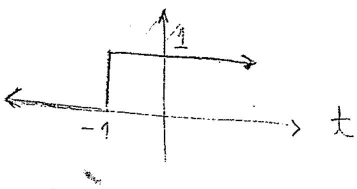
Exercícios de sinais

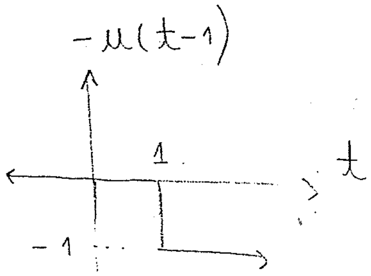
1)

a) $x(t) = rect(t/2)$



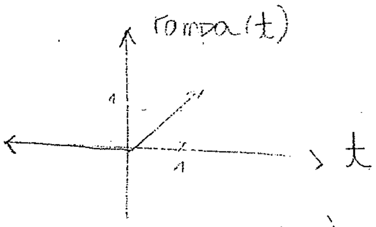


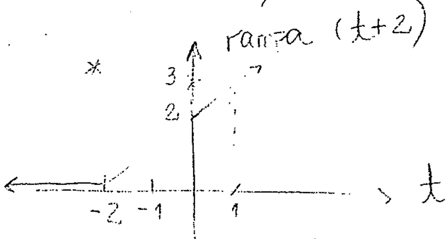


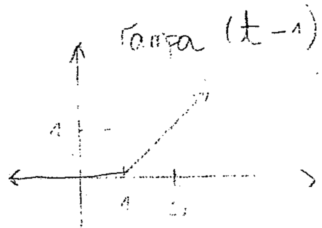


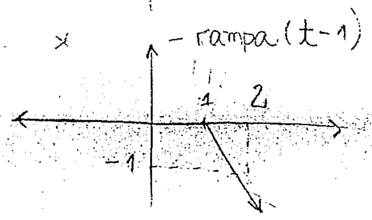
$rect(t/2) = u(t+1) - u(t-1)$

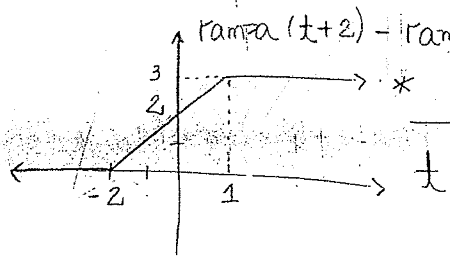
b) $x(t) = rampa(t+2) - rampa(t-1)$









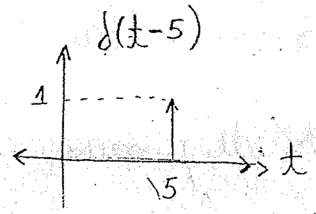


$* \quad (t+2) - (t-1) = 3$

$rampa(t) = t\,u(t) \rightarrow rampa(t+2) - rampa(t-1) =$

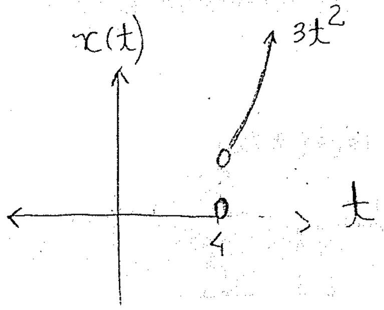
$x(t) = (t+2)\,u(t+2) - (t-1)\,u(t-1)$

c) $x(t) = \delta(t-5)$



$$\delta(t-5) = \frac{d(u(t-5))}{dt}$$

d)



$$x(t) = 3t^2 u(t-4)$$

e) $x(t) = \operatorname{sgn}(3t)$

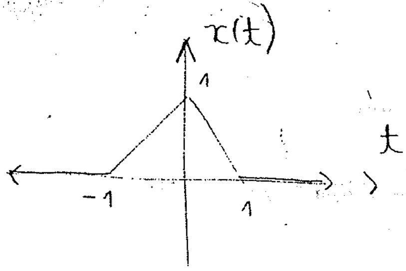
$$\operatorname{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases} \Rightarrow \operatorname{sgn}(3t) = \begin{cases} 1, & 3t > 0 \rightarrow t > 0 \\ 0 & 3t = 0 \rightarrow t = 0 \\ -1 & 3t < 0 \rightarrow t < 0 \end{cases}$$

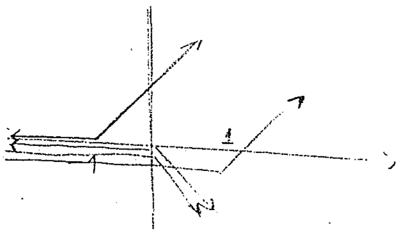
$$\operatorname{sgn}(t) = \operatorname{sgn}(3t)$$

$$u(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0 \end{cases} \qquad 2u(t) - 1 = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

$$2u(t) = \begin{cases} 2, & t > 0 \\ 1, & t = 0 \\ 0, & t < 0 \end{cases} \qquad \operatorname{sgn}(3t) = 2u(t) - 1$$

f) $x(t) = tri(t) = rampa(t+1) - 2\,rampa(t) + rampa(t-1)$

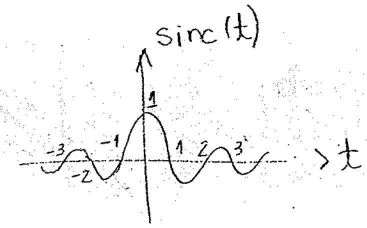


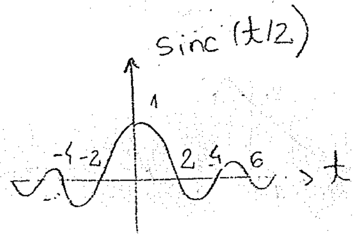


$$\therefore x(t) = tri(t) = (t+1)\,u(t+1) - 2t\,u(t) + (t-1)\,u(t-1)$$

a) $x(t) = \text{sinc}(t/2)$

$\text{sinc}(t) = \dfrac{\text{sen}(\pi t)}{\pi t}$





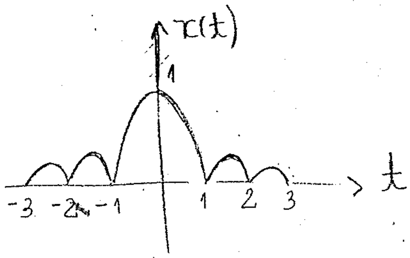
$\text{sinc}(t/2) = \dfrac{\text{sen}(\pi t/2)}{\pi t/2} = \dfrac{2\,\text{sen}(\pi t/2)}{\pi t}$ → Para $t=0$, temos que

$\lim\limits_{t\to 0} \dfrac{2\,\text{sen}(\pi t/2)}{\pi t} =$

$= 2 \lim\limits_{t\to 0} \dfrac{\pi/2\,\cos(\pi t/2)}{\pi} = 1$

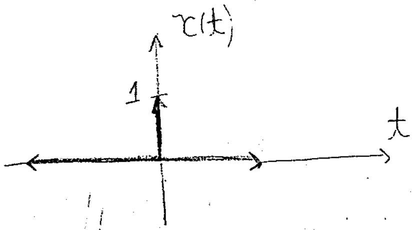
b) $x(t) = |\text{sinc}(t)|$

c)

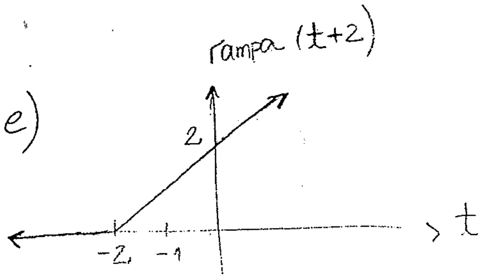


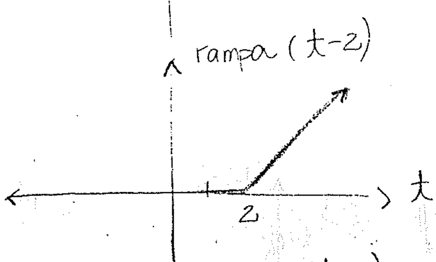
d) $x(t) = \text{dinic}(2t)$

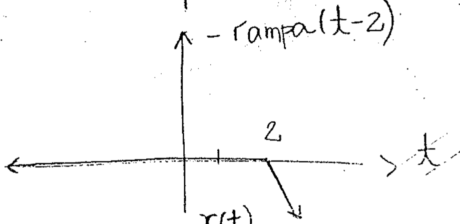
$\delta(2t) = 0, \quad 2t \neq 0, \quad t \neq 0$

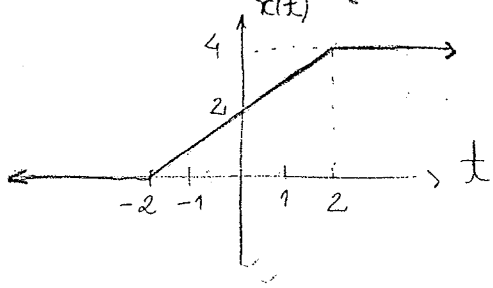


e)

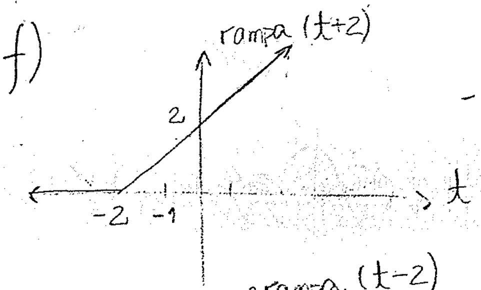


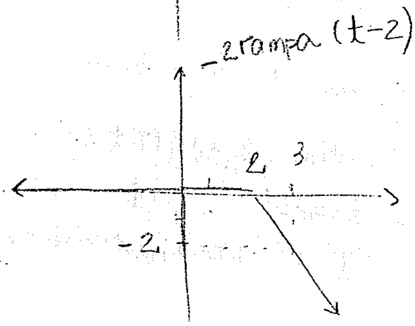




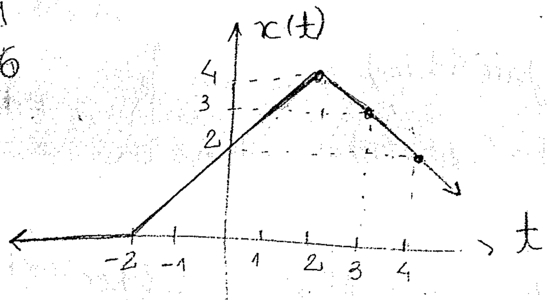


f)



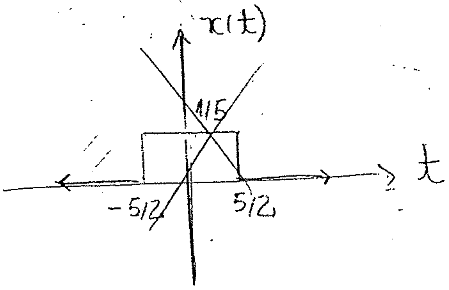


$t+2$
$-2t+4$
$-t+6$

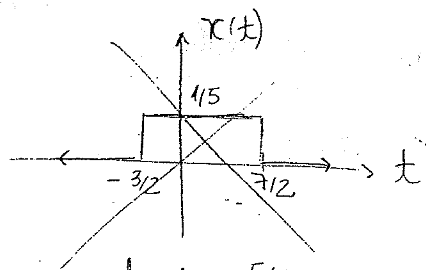


$t+2 - 2t+4 = -t+6$

g)



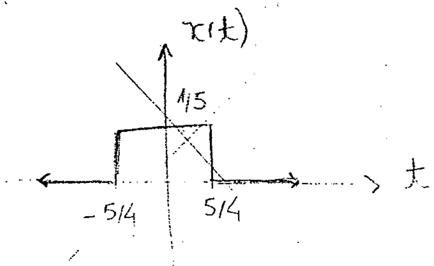
h)



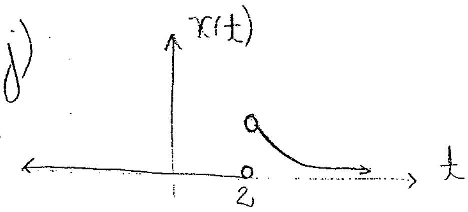
$t-1 = 5/2$

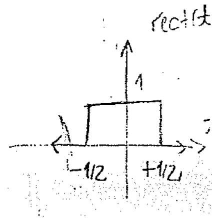
$t = 5/2 + 2/2 = 7/2$

i)

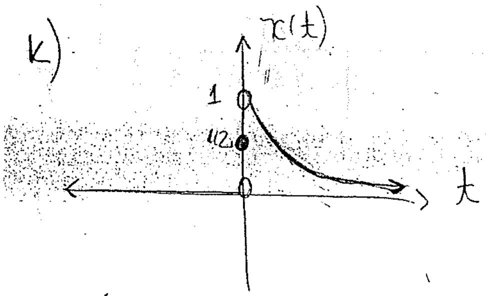


j)

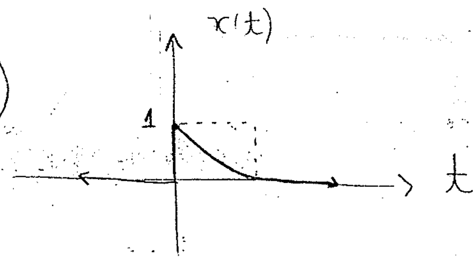




k)



l)

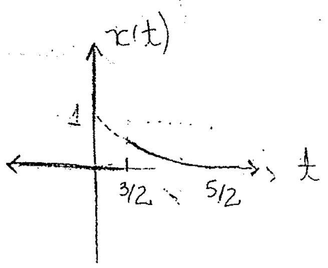


$t - 1/2 = -1/2$

$t = 0$

" abre uma passagem

m)



$t-2 = -1/2$

$t = -1/2 + 2 = 3/2$

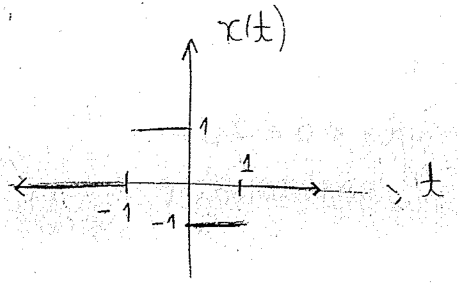
$t-2 = 1/2$

$t = 5/2$

n) $tri(t) = \begin{cases} 1-|t|, & |t|<1 \\ \phi, & |t|\geq 1 \end{cases}$

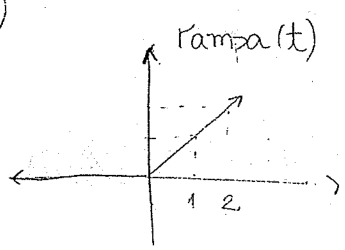
$tri(t) = \begin{cases} 1-t, & 0<t<1 \\ 1+t, & -1<t<0 \\ \phi, & t\geq 1 \text{ e } t\leq -1 \end{cases}$

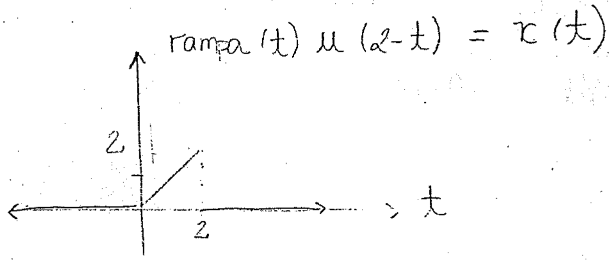
$\frac{d\,tri(t)}{dt} = \begin{cases} -1, & 0<t<1 \\ 1, & -1<t<0 \end{cases}$

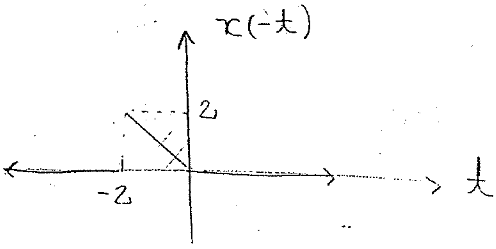

x(t)
1
1
-1
-1
t

3)

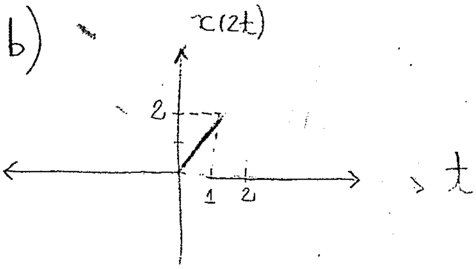
a)


rampa(t)
1
2

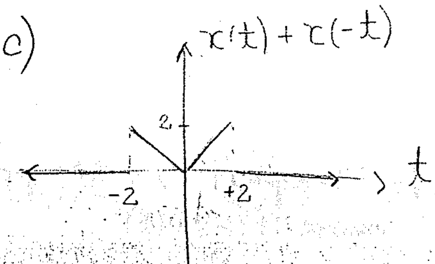

rampa(t) u(2-t) = x(t)
2
2
t


x(-t)
2
-2
t

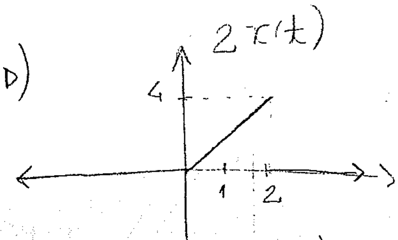
b)

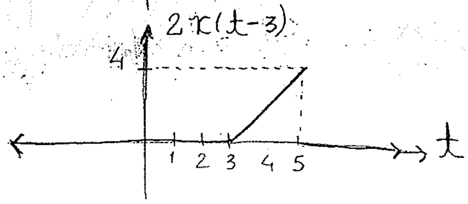

x(2t)
2
1
2
t

c)

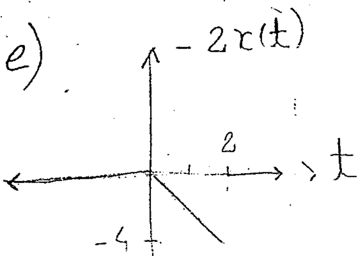

x(t) + x(-t)
2
-2
+2
t

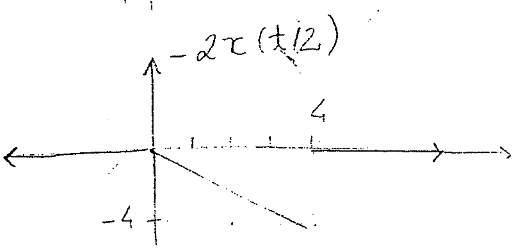
D)

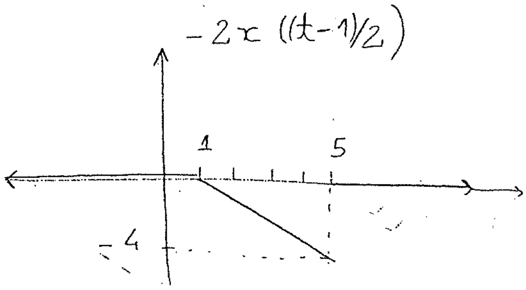

2x(t)
4
1
2


2x(t-3)
4
1 2 3 4 5
t

e)


-2x(t)
2
t
-4


-2x(t/2)
4
-4

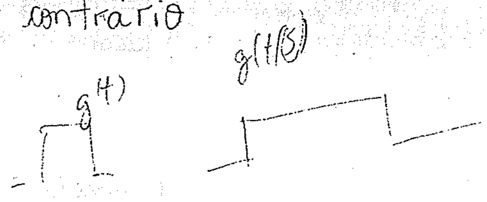

-2x((t-1)/2)
1
5
-4

4) O impulso unitário pode ser definido como:

$$\delta(t) = 0, \quad t \neq 0 \quad e \quad \int_{t_1}^{t_2} \delta(t)\,dt = \begin{cases} 1, & t_1 < 0 < t_2 \\ 0, & \text{caso contrário} \end{cases}$$

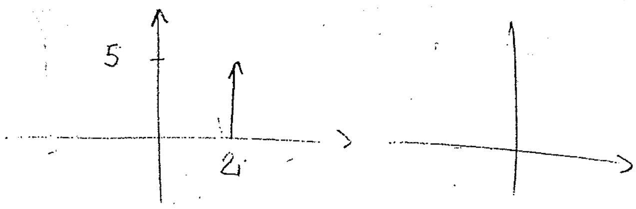
5) $\delta(t) \xrightarrow{\int} u(t) \xrightarrow{\int} \text{rampa}(t)$

$\text{rampa}(t) \xrightarrow{d/dt} u(t) \xrightarrow{d/dt} \delta(t)$



6)

a) $\displaystyle\int_{-\infty}^{\infty} \delta((t-2)/5)\,dt = 5$



$\delta(a(t-t_0)) = \dfrac{1}{|a|}\delta(t-t_0) \rightarrow \delta((t-2)/5) = 5\,\delta(t-2)$

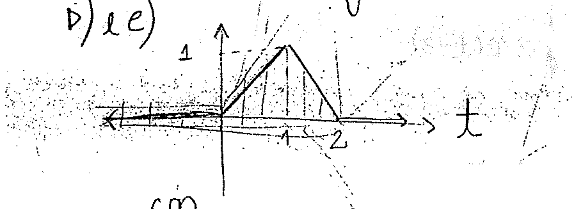
b) $\displaystyle\int_{-\infty}^{3} \delta((t-2)/5)\,dt = 5$

↳ ~~a integral inclui a localização do impulso~~

c) $\displaystyle\int_{3}^{\infty} \delta((t-2)/5)\,dt = 0$

↳ a integral não inclui a localização do impulso

d) e)



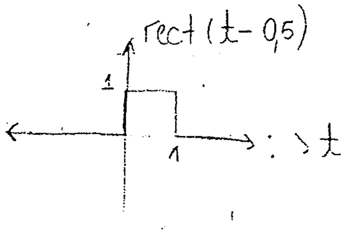
$x(t) = 1\,\text{rampa}(t) - 2\,\text{rampa}(t-1) + 1\,\text{rampa}(t-2)$

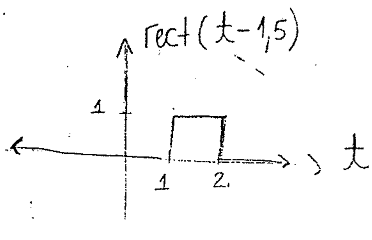
$\ast \boxed{\int_0^t x(\tau)\,d\tau}$ $\quad \text{rampa}(t) = t\,u(t)$

$= \dfrac{t^2}{2}u(t) - 2\dfrac{(t-1)^2}{2}u(t-1) + \dfrac{(t-2)^2}{2}u(t-2)$

$\displaystyle\int_{-\infty}^{\infty} \text{tri}(t-1)\,dt = \frac{2 \cdot 1}{2} = 1$ ("Área" do triângulo)

f) $\displaystyle\underbrace{\int_{-\infty}^{\infty} \text{rect}(t-0{,}5)}_{1} - 0{,}5\underbrace{\int_{-\infty}^{\infty} \text{rect}(t-1{,}5)\,dt}_{1} = 1 - 0{,}5 = 0{,}5$





7)

a) $\text{sen}(4\pi t) \rightarrow f = 2$
$\cos(12\pi t) \rightarrow f = 6$

$f_0 \cdot k = 2, \quad k \in \mathbb{Z}$
$f_0 \cdot l = 6, \quad l \in \mathbb{Z}$

$$f_0 = \frac{2}{k} = \frac{6}{l} \rightarrow \frac{k}{l} = \frac{1}{3} \in \mathbb{Q}$$

$k = 1$
$l = 3$

$\boxed{f_0 = 2}$

$\boxed{T_0 = 1/2}$

b) $\text{sen}(7\pi t) \rightarrow f = 7/2$
$\cos(8\pi t + \pi/10) \rightarrow f = 4$

$f_0 \cdot k = 7/2, \quad k \in \mathbb{Z}$
$f_0 \cdot l = 4, \quad l \in \mathbb{Z}$

$$f_0 = \frac{7/2}{k} = \frac{4}{l} \rightarrow \frac{k}{l} = \frac{7}{8} \in \mathbb{Q}$$

$f_0 = 1/2$
$\boxed{T_0 = 2}$

c) $\cos(12t) \rightarrow f = \frac{6}{\pi}$
$\cos(12\pi t) \rightarrow f = 6$

$f_0 \cdot k = 6/\pi, \quad k \in \mathbb{Z}$
$f_0 \cdot l = 6, \quad l \in \mathbb{Z}$

$$f_0 = \frac{6/\pi}{k} = \frac{6}{l} \rightarrow \frac{k}{l} = \frac{1}{\pi} \notin \mathbb{Q}$$

Sinal não é periódico

D) $x(t) = \cos(3\pi t) + j\,\text{sen}(3\pi t)$ → combinação linear de dois sinais periódicos que possuem a mesma frequência cíclica.

$$f_0 = \frac{3\pi}{2\pi} = \frac{3}{2} \rightarrow \boxed{T_0 = \frac{2}{3}}$$

8)

a) $x(t) = 4\,\text{sen}(3\pi t)$

$$x_p(t) = \frac{x(t) + x(-t)}{2} = \frac{4\,\text{sen}(3\pi t) + 4\,\text{sen}(-3\pi t)}{2} = 0$$

$$x_i(t) = \frac{x(t) - x(-t)}{2} = \frac{4\,\text{sen}(3\pi t) - 4\,\text{sen}(-3\pi t)}{2} = 4\,\text{sen}(3\pi t)$$

b) $x_i(t) = 0$

$x_p(t) = 4\cos(3\pi t)$

$\to$ função par

c) $$x_p(t) = \frac{x(t) + x(-t)}{2} = \frac{\text{rect}(t) + \text{rect}(-t)}{2} = \text{rect}(t)$$

$x_i(t) = 0$

$\frac{x(t) + x(-t)}{2}$

d) $$x_p(t) = \frac{\text{rect}(t-2) + \text{rect}(-t-2)}{2}$$

$$x_i(t) = \frac{\text{rect}(t-2) - \text{rect}(-t-2)}{2}$$

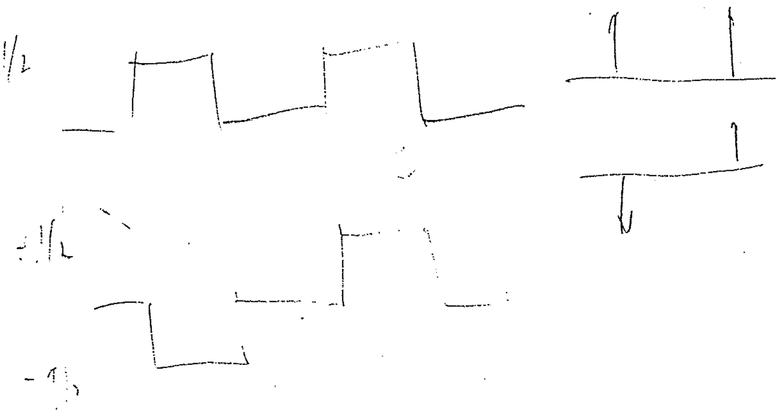
e) $$x_p(t) = \frac{\delta(t-2) + \delta(-t-2)}{2}$$

$$x_i(t) = \frac{\delta(t-2) - \delta(-t-2)}{2}$$

f) $$x_p(t) = \frac{t\cos(2\pi t) + (-t)\cos(-2\pi t)}{2} = 0$$

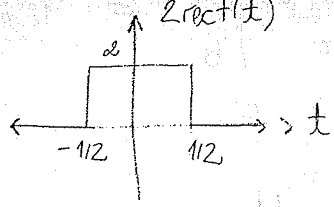
$$x_i(t) = \frac{t\cos(2\pi t) - (-t)\cos(-2\pi t)}{2} = t\cos(2\pi t)$$

(ímpar) . (par) = ímpar



9)

a) $x(t) = 2\,\text{rect}(t)$



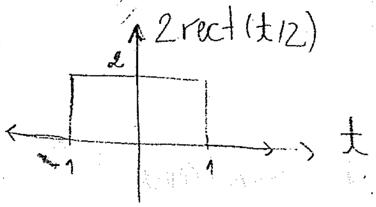
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_{0}^{1/2} |2|^2 dt = 2 \cdot \frac{1}{2} \cdot 4 = \underline{4}$$

$P_x \to$ sinal não é periódico

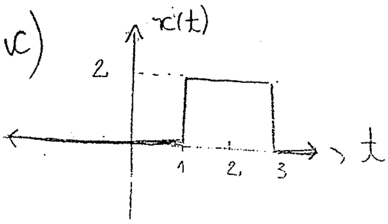
b) $x(t) = 2\,\text{rect}(t/2)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_{0}^{1} |2|^2 dt = \underline{8}$$

$P_x \to$ não periódico



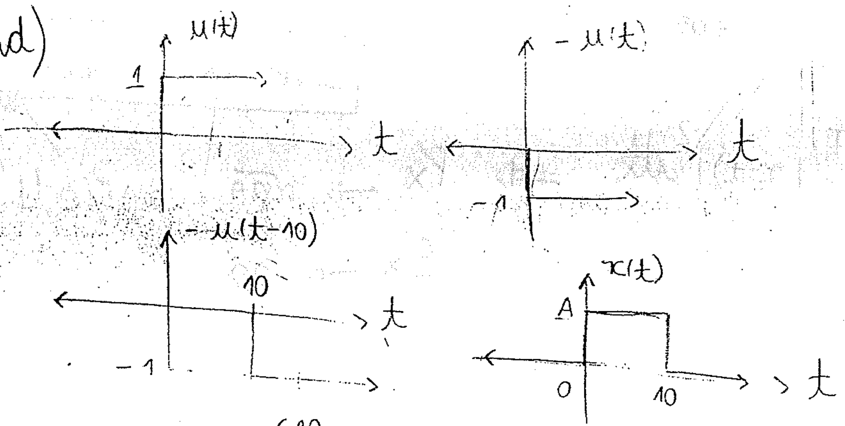
c)



$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{1}^{3} |2|^2 dt = 4 \cdot 2 = \underline{8}$$

$P_x \to$ não periódico

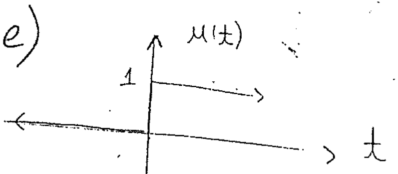
d)

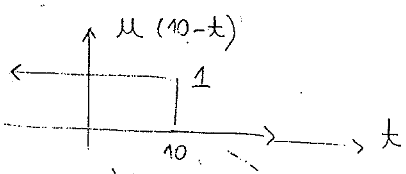


$$E_x = \int_{0}^{10} |A|^2 dt = \underline{10A^2}$$

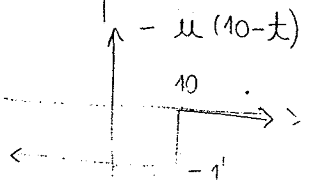
$P_x \to$ não periódico

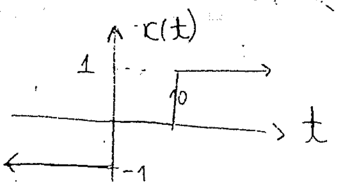
e)





$P_x \to$ não periódico





$E_x = \infty$

f) $E_x = \int_{-1/2}^{1/2} |\cos 2\pi t|^2 \, dt = 2 \int_0^{1/2} \cos^2 2\pi t \, dt =$

$= 2 \left[\frac{t}{2} + \frac{\operatorname{sen} 4\pi t}{8\pi} \right]_0^{1/2} = \left[t + \frac{\operatorname{sen} 4\pi t}{4\pi} \right]_0^{1/2} = 1/2$

$P_x \rightarrow$ não periódico

g) $E_x = \int_{-1/2}^{1/2} |\cos 4\pi t|^2 \, dt = 2 \int_0^{1/2} \cos^2 4\pi t \, dt =$

$= 2 \left[\frac{t}{2} + \frac{\operatorname{sen} 8\pi t}{16\pi} \right]_0^{1/2} = \left[t + \frac{\operatorname{sen} 8\pi t}{8\pi} \right]_0^{1/2} =$

$= 1/2$ $\qquad P_x \rightarrow$ não periódico

h) $E_x = \int_{-1/2}^{1/2} |\operatorname{sen} 2\pi t|^2 \, dt = 2 \int_0^{1/2} \operatorname{sen}^2 2\pi t \, dt =$

$= 2 \left[\frac{t}{2} - \frac{\operatorname{sen} 4\pi t}{8\pi} \right]_0^{1/2} = \left[t - \frac{\operatorname{sen} 4\pi t}{4\pi} \right]_0^{1/2} = 1/2$

$P_x \rightarrow$ não periódico

i) $E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |u(t) \operatorname{sen}(2\pi t)|^2 \, dt = \int_0^{\infty} |\operatorname{sen}(2\pi t)|^2 \, dt =$

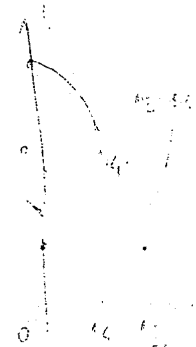
$= \infty$ $\qquad \boxed{-\infty \text{ a } +\infty}$

$P_x = \frac{1}{T} \int_{t_0}^{t_0+T} |x(t)|^2 \, dt \Longrightarrow P_x \rightarrow$ não periódico

$E_x \rightarrow \infty$

$\cos 2\pi t$

$2\pi t = \ldots, \quad 2\pi t = \frac{3\pi}{2}$



$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \infty$

$P_x = 2 \int_0^{1/4} (\cos 2\pi t)^2 \, dt$

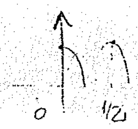
$4 \int_0^{1/4} \left[\frac{1 + \cos 4\pi t}{2} \right] dt$

$2 \left[t + \frac{1}{4\pi} \operatorname{sen} 4\pi t \right]_0^{1/4}$

$2 \left[1/4 \right] = \frac{2}{4} = \frac{1}{2}$

j) $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\cos(2\pi t)|^2 dt$, como é periódico

$E_x = \infty$ → pode-se fazer de 0 a 1/2



Sinal ao quadrado

$\boxed{T_0 = 1/2} \rightarrow \boxed{T_0/2 = 1/4}$

$$P_x = \frac{1}{1/2} \int_{-1/4}^{1/4} \cos^2(2\pi t)\, dt = 2 \cdot \cancel{2} \int_0^{1/4} \frac{1 + \cos 4\pi t}{\cancel{2}}\, dt$$

$$= 2 \left[t + \cancel{\frac{1}{4\pi} \operatorname{sen} 4\pi t} \right]_0^{1/4} = \frac{2}{4} = \frac{1}{2}$$

k) $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |e^{-2t} u(t-2)|^2 dt =$

$$= \int_2^{\infty} e^{-4t} dt = \lim_{b \to \infty} \int_2^b e^{-4t} dt = \lim_{b \to \infty} \left[-\frac{1}{4} e^{-4t} \right]_2^b =$$

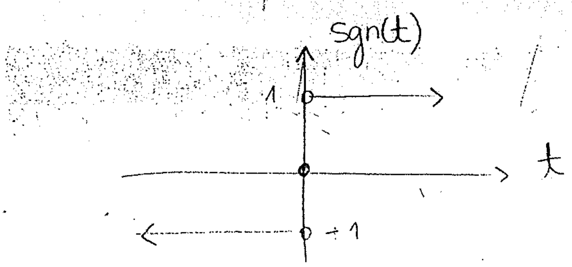
$$E_x = \frac{1}{4e^8}$$

P_x → não é periódico.

l) $\operatorname{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$

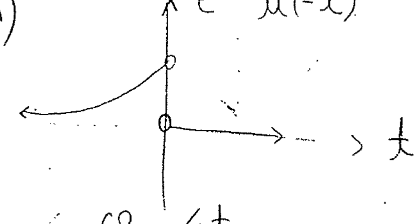
$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt =$

$= 2 \int_0^{\infty} 1\, dt = \infty$



P_x → não é periódico

m)



$$E_x = \int_{-\infty}^{0} e^{4t} dt = \frac{1}{4} e^{4t} \Big|_{-\infty}^{0} = \frac{1}{4} \left[1 - e^{-\infty} \right] = \frac{1}{4}$$

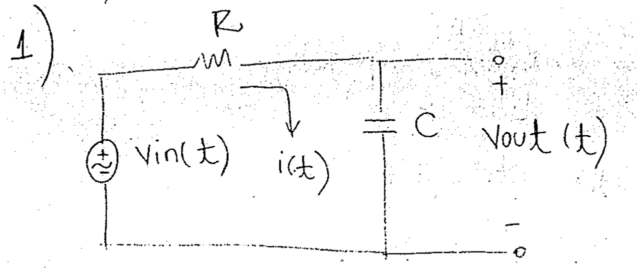
P_x → não é periódico

n) $E_x = \int_{-\infty}^{\infty} |u(t)|^2 dt = \int_0^{\infty} 1 \, dt = \infty$

P_x $\rightarrow$ não é periódico

1ª Lista - Exercícios Sistemas

1)



a) $-V_{in}(t) + R\,i(t) + V_{out}(t) = 0$

$$i(t) = C\,\frac{d\,V_{out}(t)}{dt}$$

$$-V_{in}(t) + RC\,\frac{d\,V_{out}(t)}{dt} + V_{out}(t) = 0$$

$$\Rightarrow \quad RC\,\frac{d\,V_{out}(t)}{dt} + V_{out}(t) = V_{in}(t)$$

b) Solução homogênea:

$$RC\,\lambda + 1 = 0$$

$$\lambda = -\frac{1}{RC}$$

$$\Rightarrow \quad V_{out_h}(t) = K_h\,e^{-t/RC}$$

c) Solução Particular: $V_{in}(t) = A \;,\; t > 0$

$$V_{out_p}(t) = K_P$$

$$K_P = A$$

$$V_{out}(t) = K_h e^{-t/RC} + A \;,\; t > 0$$

Para $t = 0$, $V_{out}(0) = 0$

$$0 = K_h + A \Rightarrow K_h = -A$$

$$\Rightarrow \quad \therefore\; V_{out}(t) = A\left(1 - e^{-t/RC}\right) \;,\; t > 0$$

$$V_{out}(t) = A\left(1 - e^{-t/RC}\right) u(t)$$

d) Linearidade: $a\,F\{x_1(t), y_1(t)\} + b\,F\{x_2(t), y_2(t)\} = F\{a x_1(t) + b x_2(t), a y_1(t) + b y_2(t)\}$

$$F\{V_{in}(t), V_{out}(t)\} = RC \frac{d V_{out}(t)}{dt} + V_{out}(t) - V_{in}(t)$$

Tomando:

$F\{V_{in_1}(t), V_{out_1}(t)\}$ e $F\{V_{in_2}(t), V_{out_2}(t)\}$

Para a linearidade:

$$\underbrace{a\,F\{V_{in_1}(t), V_{out_1}(t)\} + b\,F\{V_{in_2}(t), V_{out_2}(t)\}}_{①} = \underbrace{F\{a V_{in_1}(t) + b V_{in_2}(t), a V_{out_1}(t) + b V_{out_2}(t)\}}_{②}$$

① $a RC \frac{d V_{out_1}(t)}{dt} + a V_{out_1}(t) - a V_{in_1}(t) + b RC \frac{d V_{out_2}(t)}{dt} + b V_{out_2}(t) - b V_{in_2}(t)$

② $RC \frac{d}{dt}\{a V_{out_1}(t) + b V_{out_2}(t)\} + a V_{out_1}(t) + b V_{out_2}(t) - a V_{in_1}(t) - b V_{in_2}(t)$

⇒ como ① é igual a ② o sistema é linear.

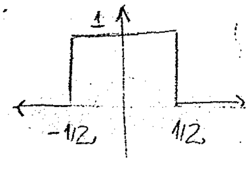
e) O sistema é causal, visto que este, com condições iniciais nulas, apresenta resposta somente durante ou após a aplicação da excitação.

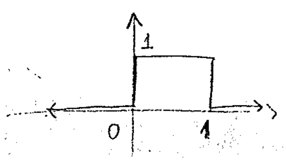
f) $G(t) = F\{V_{in}(t), V_{out}(t)\} = RC \frac{d V_{out}(t)}{dt} + V_{out}(t) - V_{in}(t)$

$G_1(t) = F\{V_{in}(t-t_0), V_{out}(t-t_0)\} = RC \frac{d V_{out}(t-t_0)}{dt} + V_{out}(t-t_0) - V_{in}(t-t_0)$

$G(t-t_0) = G_1(t)$ ⇒ logo o sistema é invariante no tempo.

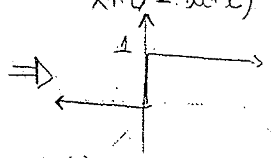
2) rect(t) $\qquad$ $x(t) = rect(t-0{,}5)$





⇒

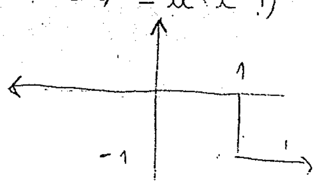
$x_1(t) = u(t)$



⇒ $x_1(t) = u(t)$

$y_1(t) = (1 - e^{-t/RC})\, u(t)$

$x_2(t) = -u(t-1)$



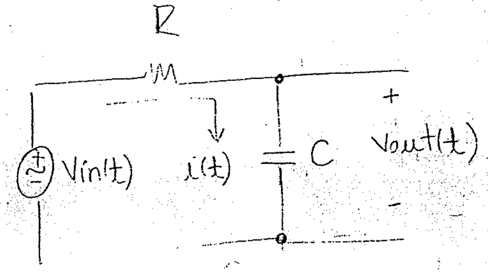
$x_2(t) = -u(t-1)$

$y_2(t) = -(1 - e^{-(t-1)/RC})\, u(t-1)$

$x(t) = x_1(t) + x_2(t)$ $\qquad$ $y(t) = (1 - e^{-t/RC})\, u(t) - (1 - e^{-(t-1)/RC})\, u(t-1)$

3)

a)



$$-V_{in}(t) + R\,i(t) + V_{out}(t) = 0$$

$$RC\frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t)$$

Solução homogênea:

$$RC\lambda + 1 = 0$$

$$\lambda = -1/RC$$

$$V_{outh}(t) = K_h e^{-t/RC}$$

Solução particular:

$$V_{in}(t) = A, \quad t > 0$$

$$V_{outp}(t) = K_p$$

$$\Rightarrow K_p = A \quad \Rightarrow V_{outp}(t) = A$$

$$V_{out}(t) = K_h e^{-t/RC} + A$$

$$V_{out}(0) = B$$

$$B = K_h + A$$

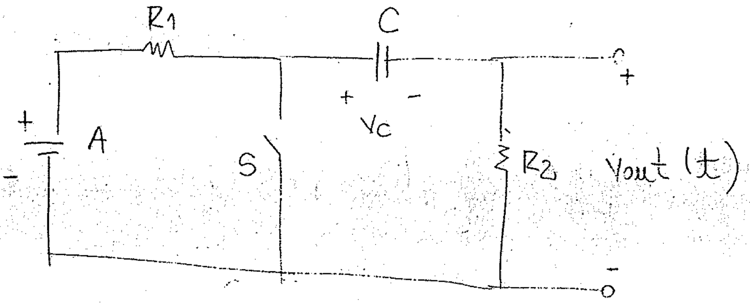
$$K_h = B - A$$

$$V_{out}(t) = (B - A)e^{-t/RC} + A, \quad t > 0$$

$$V_{out}(t) = \left[A(1 - e^{-t/RC}) + Be^{-t/RC}\right]u(t)$$

c)

4)

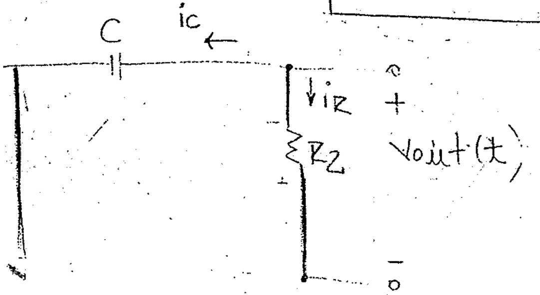


a) $t < 0$

$V_C(0^-) = A$

$\boxed{t > 0}$

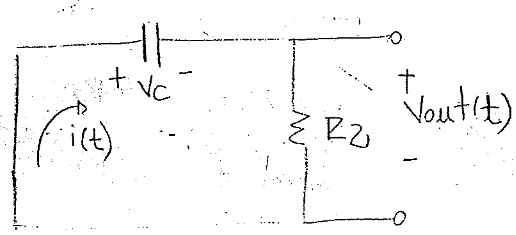
$V_C(0^+) = A \rightarrow \boxed{V_{out}(0) = -A}$



$i_C + i_R = 0$

$$C \frac{d V_{out}(t)}{dt} + \frac{V_{out}(t)}{R_2} = 0$$

$$\Rightarrow R_2 C \frac{d V_{out}(t)}{dt} + V_{out}(t) = 0 \quad t > 0$$



(*) $V_C + V_{out}(t) = 0$

$$\frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau + v_{out}(t) = \emptyset$$

$$\frac{1}{C}\int \frac{V_{out}(\tau)}{R_2}\,d\tau + v_{out}(t) = \emptyset$$

$$\int_{-\infty}^{t} V_{out}(\tau)\,d\tau + R_2 C V_{out} = \emptyset$$

$$R_2 C \int_{-\infty}^{t} \frac{d V_{out}(\tau)}{d\tau}\,d\tau + \ldots + R_2 C \frac{d V_{out}(t)}{dt} = 0$$

$$\Rightarrow R_2 C \frac{d V_{out}(t)}{dt} + V_{out}(t) = 0$$

b)

$R_2 C \lambda + 1 = 0$

$\lambda = -1/R_2 C$

$V_{out}(t) = K e^{-t/R_2 C}$

$V_{out}(0) = -A$

$$\Rightarrow V_{out}(t) = \left[-A e^{-t/R_2 C}\right] u(t)$$

c) $F\{x(t), y(t)\} = R_2 C \frac{d\,V_{out}(t)}{dt} + V_{out}(t)$ $\quad \underline{x(t) = 0}$

Tomando:

$F\{x_1(t), y_1(t)\}$ e $F\{x_2(t), y_2(t)\}$

Linearidade: $a F\{x_1(t), y_1(t)\} + b F\{x_2(t), y_2(t)\} =$ (1)

$= F\{a x_1(t) + b x_2(t), a y_1(t) + b y_2(t)\}$ (2)

(1) $a R_2 C \frac{d\,V_{out_1}(t)}{dt} + a V_{out_1}(t) + b R_2 C \frac{d\,V_{out_2}(t)}{dt} + b V_{out_2}(t)$

(2) $R_2 C \frac{d}{dt}\{a V_{out_1}(t) + b V_{out_2}(t)\} + a V_{out_1}(t) + b V_{out_2}(t)$

(1) = (2) $\Rightarrow$ Linear

D)

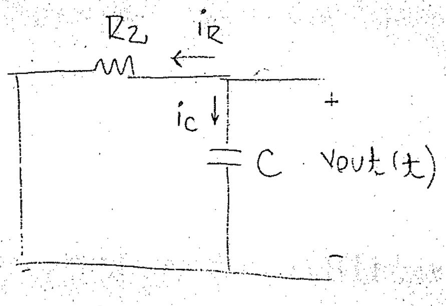
e) $G(t) = F\{x(t), y(t)\} = R_2 C \frac{d\,V_{out}(t)}{dt} + V_{out}(t)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\} = R_2 C \frac{d\,V_{out}(t-t_0)}{dt} + V_{out}(t-t_0)$

$G_1(t) = G(t-t_0) \Rightarrow$ Invariante no tempo

5)

a) $\boxed{V_{out}(0) = A}$



$$i_R + i_C = 0$$

$$\frac{V_{out}(t)}{R_2} + C\frac{dV_{out}(t)}{dt} = 0$$

$$\Rightarrow R_2C\frac{dV_{out}(t)}{dt} + V_{out}(t) = 0 \quad , \quad t>0$$

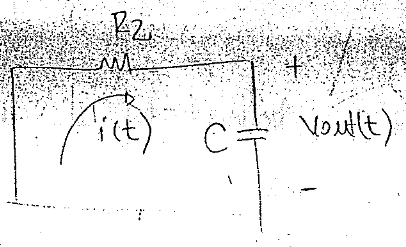
b) $R_2C\lambda + 1 = 0$

$$\lambda = -1/R_2C$$

$$V_{out}(t) = K e^{-t/R_2C}$$

$$V_{out}(0) = A$$

$$\Rightarrow V_{out}(t) = \left[A e^{-t/R_2C}\right] u(t)$$



$$R_2 i(t) + V_{out}(t) = 0$$

$$R_2C\frac{dV_{out}(t)}{dt} + V_{out}(t) = 0 \quad , t>0$$

$$R_2C\lambda + 1 = 0 \Rightarrow \lambda = -1/R_2C$$

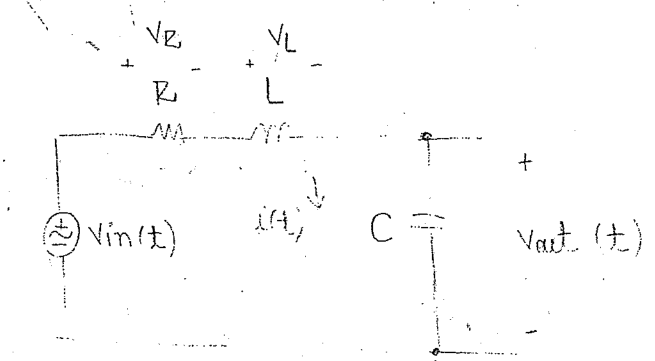
$$V_{outh}(t) = K e^{-t/R_2C}$$

6)

a) $\boxed{V_{out}(t) = V_{in}(t)}$

$\underline{y(t) = x(t)}$

7)



a) $- V_{in}(t) + R\,i(t) + L\dfrac{di(t)}{dt} + V_{out}(t) = 0$

$$- V_{in}(t) + RC\frac{d\,V_{out}(t)}{dt} + L\frac{d}{dt}\left(C\frac{dV_{out}(t)}{dt}\right) + V_{out}(t) = 0$$

$$LC\frac{d^2 V_{out}(t)}{dt^2} + RC\frac{d\,V_{out}(t)}{dt} + V_{out}(t) = V_{in}(t)$$

$$\Rightarrow \frac{d^2 V_{out}(t)}{dt^2} + \frac{R}{L}\frac{d\,V_{out}(t)}{dt} + \frac{1}{LC}V_{out}(t) = \frac{1}{LC}V_{in}(t)$$

b) Solução homogênea:

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$$

$$\lambda = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

$$\lambda = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$V_{out\,h}(t) = K_1 e^{\left(-R/2L + \sqrt{(R/2L)^2 - 1/LC}\right)t} + K_2\, e^{\left(-R/2L - \sqrt{(R/2L)^2 - 1/LC}\right)t}$$

c) Solução particular: $V_{in}(t) = A,\ t > 0$

$$V_{out\,p}(t) = K_p$$

$$\cancel{\frac{1}{LC}} K_p = \frac{1}{\cancel{LC}} A$$

$$K_p = A \Rightarrow V_{out\,p}(t) = A$$

$$\therefore V_{out}(t) = \left[K_1 e^{\left(-R/2L + \sqrt{(R/2L)^2 - 1/LC}\right)t} + K_2\, e^{\left(-R/2L - \sqrt{(R/2L)^2 - 1/LC}\right)t} + A\right] u(t)$$

D) $$F\{x(t), y(t)\} = \frac{d^2 V_{out}}{dt^2} + \frac{R}{L}\frac{dV_{out}}{dt} + \frac{1}{LC}V_{out} - \frac{1}{LC}V_{in}$$

$$\underbrace{a F\{x_1(t), y_1(t)\} + b F\{x_2(t), y_2(t)\}}_{1} = F\underbrace{\{a x_1(t) + b x_2(t),\ a y_1(t) + b y_2(t)\}}_{2}$$

1 $$a \frac{d^2 \cancel{V_{out_1}}}{\cancel{dt^2}} + a\frac{R}{L}\frac{d \cancel{V_{out_1}}}{\cancel{dt}} + \frac{a}{\cancel{LC}} V_{out_1} - \frac{a}{LC} V_{in_1} + b\frac{d^2 \cancel{V_{out_2}}}{\cancel{dt^2}} + \frac{bR}{\cancel{L}}\frac{d V_{out_2}}{dt}$$
$$+ \frac{b}{LC} \cancel{V_{out_2}} - \frac{b}{\cancel{LC}} V_{in_2}$$

2 $$\frac{d^2}{dt^2}\{a \cancel{V_{out_1}} + b \cancel{V_{out_2}}\} + \frac{R}{L}\frac{d}{dt}\{\cancel{aV_{out_1}} + \cancel{bV_{out_2}}\} + \frac{1}{LC}\{a \cancel{V_{out_1}} + b \cancel{V_{out_2}}\} -$$
$$- \frac{1}{LC}\{a \cancel{V_{in_1}} + b V_{in_2}\}$$

1 = 2 é linear

8)

a) $y(t) - 1 = x(t)$

$$F\{x(t), y(t)\} = y(t) - x(t) - 1$$

$$\underbrace{aF\{x_1(t), y_1(t)\} + bF\{x_2(t), y_2(t)\}}_{①} = F\{\underbrace{ax_1(t) + bx_2(t), ay_1(t) + by_2(t)}_{②}\}$$

① $a y_1(t) - a x_1(t) - a + b y_2(t) - b x_2(t) - b$

② $a y_1(t) + b y_2(t) - a x_1(t) - b x_2(t) - 1$

$-a - b \neq -1$

① $\neq$ ②, logo é Não-Linear

b) $y(t) = x(t)$

$$F\{x(t), y(t)\} = y(t) - x(t)$$

$$\underbrace{aF\{x_1(t), y_1(t)\} + bF\{x_2(t), y_2(t)\}}_{①} = F\{\underbrace{ax_1(t) + bx_2(t), ay_1(t) + by_2(t)}_{②}\}$$

① $a y_1(t) - a x_1(t) + b y_2(t) - b x_2(t)$

② $a y_1(t) + b y_2(t) - a x_1(t) - b x_2(t)$

① = ②, logo é Linear

c) $\frac{dy(t)}{dt} + a\,y(t) = x(t)$

$$F\{x(t), y(t)\} = \frac{dy(t)}{dt} + a\,y(t) - x(t)$$

$$\underbrace{a\,F\{x_1(t), y_1(t)\} + b\,F\{x_2(t), y_2(t)\}}_{①} = F\{\underbrace{a x_1(t) + b x_2(t),\ a y_1(t) + b y_2(t)}_{②}\}$$

① $a\frac{dy_1(t)}{dt} + a^2 y_1(t) - a x_1(t) + b\frac{dy_2(t)}{dt} + ab y_2(t) - b x_2(t)$

② $\frac{d}{dt}\{a y_1(t) + b y_2(t)\} + a\{a y_1(t) + b y_2(t)\} - a x_1(t) - b x_2(t)$

① = ② , Logo é Linear

e) $F\{x(t), y(t)\} = \int_{-\infty}^{t} y(\tau)\,d\tau + a\,y(t) - \frac{d}{dt}x(t)$

$$\underbrace{a\,F\{x_1(t), y_1(t)\} + b\,F\{x_2(t), y_2(t)\}}_{①} = F\{\underbrace{a x_1(t) + b x_2(t),\ a y_1(t) + b y_2(t)}_{②}\}$$

① $a\left\{\int_{-\infty}^{t} y_1(\tau)\,d\tau\right\} + a^2 y_1(t) - a\frac{d}{dt}x_1(t) + b\left\{\int_{-\infty}^{t} y_2(\tau)\,d\tau\right\} + ab y_2(t) - b\frac{d x_2(t)}{dt}$

② $\int_{-\infty}^{t}\{a y_1(t) + b y_2(t)\}d\tau + a\{a y_1(t) + b y_2(t)\} - \frac{d}{dt}\{a x_1(t) + b x_2(t)\}$

① = ② , Logo é Linear

f) $F\{x(t), y(t)\} = y(t) - u(x(t))$

Para ser linear:

$$\underbrace{a\,F\{x_1(t), y_1(t)\} + b\,F\{x_2(t), y_2(t)\}}_{①} = F\{\underbrace{a x_1(t) + b x_2(t), a y_1(t) + b y_2(t)}_{②}\}$$

① $\cancel{a y_1(t)} - a u(x_1(t)) + b y_2(t) - b u(x_2(t))$

② $\cancel{a y_1(t)} + b y_2(t) - u(a x_1(t) + b x_2(t))$

$$- a u(x_1(t)) - b u(x_2(t)) \neq - u(a x_1(t) + b x_2(t))$$

① ≠ ②, logo não é linear

9)

a) $G(t) = F\{x(t), y(t)\} = \frac{dy(t)}{dt} + a y(t) + b - x(t)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\} = \frac{dy(t-t_0)}{dt} + a y(t-t_0) + b - x(t-t_0)$

$\Rightarrow G(t-t_0) = G_1(t)$, logo é invariante

b) $G(t) = F\{x(t), y(t)\} = \frac{dy(t)}{dt} + a\,\text{sen}(y(t)) - x(t)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\} = \frac{dy(t-t_0)}{dt} + a\,\text{sen}(y(t-t_0)) - x(t-t_0)$

$G(t-t_0) = G_1(t)$, logo é invariante

c) $G(t) = F\{x(t), y(t)\} = t\frac{dy(t)}{dt} - a y(t) - x(t)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\} = t\frac{dy(t-t_0)}{dt} - a y(t-t_0) - x(t-t_0)$

$G(t-t_0) = (t-t_0)\frac{dy(t-t_0)}{dt} - a y(t-t_0) - x(t-t_0)$

$G_1(t) \neq G(t-t_0)$, logo é variante

d) $G(t) = F\{x(t), y(t)\} = y(t) - 1 - x(t)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\} = y(t-t_0) - 1 - x(t-t_0)$

$G(t-t_0) = G_1(t)$, invariante

e) $G(t) = F\{x(t), y(t)\} = y(t) - \cos(2\pi t)\, x(t)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\} = y(t-t_0) - \cos(2\pi t)\, x(t-t_0)$

$G(t-t_0) = y(t-t_0) - \cos(2\pi(t-t_0))\, x(t-t_0)$

$G(t-t_0) \neq G_1(t)$, logo é variante

$G(t) = F\{x(t), y(t)\} \rightarrow G(t-t_0)$

$G_1(t) = F\{x(t-t_0), y(t-t_0)\}$

$G_1(t) = G(t-t_0) \rightarrow$ Invariante

$F\{x(t), y(t)\}$

$$\underbrace{a F\{x_1(t), y_1(t)\} + b F\{x_2(t), y_2(t)\}}_{1} = F\{\underbrace{a x_1(t) + b x_2(t), a y_1(t) + b y_2(t)}_{2}\}$$

1ª Lista - Exercícios de Convolução

1)

a) $y^{\overset{(n)}{'}}(t) + 5y(t) = \overset{(m)}{x}(t)$

$$\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$$

$$\boxed{h'(t) + 5h(t) = \delta(t)}$$

Solução homogênea:

$\lambda + 5 = 0$

$\lambda = -5$

$h_h(t) = Ae^{-5t}u(t)$

Solução total

como $n > m$, temos:

$h(t) = Ae^{-5t}u(t)$

Integrando a EDO de 0_- a 0_+:

$$\int_{0^-}^{0^+} h'(t)\,dt + 5\int_{0^-}^{0^+} h(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt$$

$h(0^+) - h(0^-) + \emptyset = 1$

$A - \emptyset + \emptyset = 1 \Rightarrow A = 1$

$$\therefore h(t) = e^{-5t}u(t)$$

b) $\overset{(n)}{y''}(t) + 6y'(t) + 4y(t) = \overset{(m)}{x}(t)$

$$\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$$

$$\boxed{h''(t) + 6h'(t) + 4h(t) = \delta(t)}$$

Solução homogênea

$\lambda^2 + 6\lambda + 4 = 0$

$\Delta = 36 - 4.1.4 = 36 - 16 = 20$

$\lambda = \dfrac{-6 \pm \sqrt{20}}{2}$ $\begin{cases} \lambda_1 = -0{,}764 \\ \lambda_2 = -5{,}236 \end{cases}$

$h_h(t) = Ae^{-0{,}764t} + Be^{-5{,}236t}$

Solução total

$n > m$. $h(t) = [Ae^{-0{,}764t} + Be^{-5{,}236t}]u(t)$

Integrando a EDO de 0_- a 0_+:

$$\int_{0^-}^{0^+} h''(t)\,dt + 6\int_{0^-}^{0^+} h'(t)\,dt + 4\int_{0^-}^{0^+} h(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt$$

$h'(t)\Big|_{0^-}^{0^+} + 6h(t)\Big|_{0^-}^{0^+} + 0 = 1$

$-0{,}764A - 5{,}236B + 6A + 6B = 1$

$5{,}236A + 0{,}764B = 1 \quad (1)$

Integrando a EDO de $-\infty$ a t:

$$\int_{-\infty}^{t} h''(\tau)\,d\tau + 6\int_{-\infty}^{t} h'(\tau)\,d\tau + 4\int_{-\infty}^{t} h(\tau)\,d\tau = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

$h'(t) + 6h(t) + 4\int_{-\infty}^{t} h(\tau)\,d\tau = u(t)$

Integrando de 0_- a 0_+:

$$\int_{0_-}^{0^+} h'(t)\,dt + 6\int_{0_-}^{0^+} h(t)\,dt + 4\int_{0_-}^{0^+}\left[\int_{-\infty}^{t} h(\tau)\,d\tau\right]dt = 0$$

$A + B = 0 \quad (2)$

$$\begin{cases} 5{,}236A + 0{,}764B = 1 \Rightarrow 5{,}236A - 0{,}764A = 1 \Rightarrow A = 0{,}2236 \\ A + B = 0 \Rightarrow B = -A \qquad\qquad B = -0{,}2236 \end{cases}$$

$$\therefore h(t) = \left[0{,}2236\, e^{-0{,}764t} - 0{,}2236\, e^{-5{,}236t}\right] u(t)$$

c) $2y'(t) + 3y(t) = x'(t)$

$y'(t) + 1{,}5\, y(t) = 0{,}5\, x'(t)$

$$\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$$

EDO:

$h'(t) + 1{,}5\, h(t) = 0{,}5\, \delta'(t)$

Solução homogênea:

$\lambda + 1{,}5 = 0$

$\lambda = -1{,}5$

$h_h(t) = K e^{-1{,}5t}$

Solução total:

$n = m : \; h(t) = K e^{-1{,}5t} u(t) + B\,\delta(t)$

$$\int_{0^-}^{0^+} h'(t)\,dt + 1{,}5 \int_{0^-}^{0^+} h(t)\,dt = 0{,}5 \int_{0^-}^{0^+} \delta'(t)\,dt$$

$$h(0^+) - h(0^-) + 1{,}5 \,.\, B = 0{,}5\, \underbrace{(\delta(0^+) - \delta(0^-))}_{\emptyset}$$

$K + 1{,}5B = 0 \quad (1)$

$$\int_{-\infty}^{t} h'(\tau)\,d\tau + 1{,}5 \int_{-\infty}^{t} h(\tau)\,d\tau = 0{,}5 \int_{-\infty}^{t} \delta'(\tau)\,d\tau$$

$$h(t) + 1{,}5 \int_{-\infty}^{t} h(\tau)\,d\tau = 0{,}5\, \delta(t)$$

$$\int_{0^-}^{0^+} h(t)\,dt + 1{,}5 \int_{0^-}^{0^+} \int_{-\infty}^{t} h(\tau)\,d\tau\,dt = 0{,}5 \int_{0^-}^{0^+} \delta(t)\,dt$$

$B + \emptyset = 0{,}5 \Rightarrow B = 0{,}5$

$K = -0{,}75$

$$\therefore h(t) = -0{,}75 e^{-1{,}5t} u(t) + 0{,}5\, \delta(t)$$

//

D) $4y''(t) + 9y'(t) = x'(t) \Rightarrow y''(t) + 2{,}25\,y'(t) = 0{,}25\,x'(t)$

$$\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$$

EDO:

$$h''(t) + 2{,}25\,h'(t) = 0{,}25\,\delta'(t)$$

Solução homogênea:

$$\lambda^2 + 2{,}25\lambda = 0$$

$$\lambda(\lambda + 2{,}25) = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = -2{,}25$$

$$h_h(t) = A + B e^{-2{,}25t}\,u(t)$$

Solução total:

$n > m$:

$$h(t) = [A + B e^{-2{,}25t}]\,u(t)$$

$$\int_{0^-}^{0^+} h''(t)\,dt + 2{,}25\int_{0^-}^{0^+} h'(t)\,dt = 0{,}25\int_{0^-}^{0^+} \delta'(t)\,dt$$

$$h'(t)\Big|_{0^-}^{0^+} + 2{,}25\,h(t)\Big|_{0^-}^{0^+} = 0{,}25\,(\delta(0^+) - \delta(0^-))$$

$$-\cancel{2{,}25B} + \cancel{2{,}25B} + 2{,}25A = 0$$

$$A = 0$$

$$\int_{-\infty}^{t} h''(\tau)\,d\tau + 2{,}25\int_{-\infty}^{t} h'(\tau)\,d\tau = 0{,}25\int_{-\infty}^{t} \delta'(\tau)\,d\tau$$

$$h'(t) + 2{,}25\,h(t) = 0{,}25\,\delta(t)$$

$$\int_{0^-}^{0^+} h'(t)\,dt + 2{,}25\int_{0^-}^{0^+} h(t)\,dt = 0{,}25\int_{0^-}^{0^+} \delta(t)\,dt$$

$$h(t)\Big|_{0^-}^{0^+} + \emptyset = 0{,}25$$

$$A + B = 0{,}25 \Rightarrow B = 0{,}25$$

$$\therefore\ h(t) = [0{,}25\,e^{-2{,}25t}]\,u(t)$$

2) $4\,y''(t) = 2\,x(t) - x'(t)$

$$y''(t) = 0{,}5\,x(t) - 0{,}25\,x'(t)$$

$$\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$$

EDO: $h''(t) = 0{,}5\,\delta(t) - 0{,}25\,\delta'(t)$

Solução homogênea:

$$\left.\begin{array}{l} \lambda^2 = 0 \\ \lambda_1 = 0 \\ \lambda_2 = 0 \end{array}\right\} \; h_h(t) = (A + Bt)\,e^{0t} = (A + Bt)\,u(t)$$

Solução completa:

$n > m$:

$$\boxed{h(t) = [A + Bt]\,u(t)}$$

$$\int_{0^-}^{0^+} h''(t)\,dt = 0{,}5\int_{0^-}^{0^+} \delta(t)\,dt - 0{,}25\int_{0^-}^{0^+} \delta'(t)\,dt$$

$$h'(t)\Big|_{0^-}^{0^+} = 0{,}5 - 0{,}25\,\underbrace{(\delta(0^+) - \delta(0^-))}_{\emptyset}$$

$$B = 0{,}5$$

$$\int_{-\infty}^{t} h''(\tau)\,d\tau = 0{,}5\int_{-\infty}^{t} \delta(\tau)\,d\tau - 0{,}25\int_{-\infty}^{t} \delta'(\tau)\,d\tau$$

$$h'(t) = 0{,}5\,u(t) - 0{,}25\,\delta(t)$$

$$\int_{0^-}^{0^+} h'(t)\,dt = 0{,}5\,\underbrace{\cancel{\int_{0^-}^{0^+} u(t)\,dt}}_{\emptyset} - 0{,}25\int_{0^-}^{0^+} \delta(t)\,dt$$

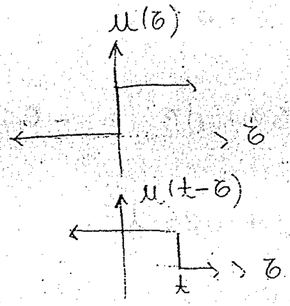
$$\underbrace{h(t)\Big|_{0^-}^{0^+}}_{} = -0{,}25$$

$$A = -0{,}25$$

$$\therefore\; h(t) = [-0{,}25 + 0{,}5\,t]\,u(t)$$

2)

a) $h_u(t) = h(t) * u(t)$



$$h_u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$h_u(t) = \int_{-\infty}^{\infty} e^{-5\tau} u(\tau)\, u(t-\tau)\, d\tau$$

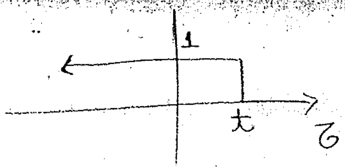
$$h_u(t) = \int_0^t e^{-5\tau} d\tau = -\frac{1}{5} e^{-5\tau} \Big|_0^t = -\frac{1}{5}\left(e^{-5t} - 1\right) = \frac{1}{5}\left(1 - e^{-5t}\right) u(t)$$

$$\Rightarrow h_u(t) = \left[\frac{1}{5}\left(1 - e^{-5t}\right)\right] u(t)$$

c)

$h_u(t) = h(t) * u(t)$

$$h_u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$



$$h_u(t) = \int_{-\infty}^{\infty} \left[-0{,}75\, e^{-1{,}5\tau} u(\tau) + 0{,}5\, \delta(\tau)\right] u(t-\tau)\, d\tau$$

$$h_u(t) = \int_{-\infty}^{\infty} -0{,}75\, e^{-1{,}5\tau} u(\tau)\, u(t-\tau)\, d\tau + \underbrace{\int_{-\infty}^{\infty} 0{,}5\, \delta(\tau)\, u(t-\tau)\, d\tau}_{\text{verificar!}}$$

$$h_u(t) = \int_0^t -0{,}75\, e^{-1{,}5\tau} d\tau + 0{,}5\, u(t)$$

$$\int_{-\infty}^{t} 0{,}5\, \delta(\tau)\, d\tau$$

$$h_u(t) = 0{,}5\, e^{-1{,}5\tau} \Big|_0^t + 0{,}5\, u(t)$$

$$\Rightarrow h_u(t) = 0{,}5\, e^{-1{,}5t} u(t) - 0{,}5\, u(t) + 0{,}5\, u(t)$$

$$\Rightarrow h_u(t) = 0{,}5\, e^{-1{,}5t}$$

D) $h_u(t) = h(t) * u(t)$

$$h_u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$h_u(t) = \int_0^t 0{,}25\, e^{-2{,}25\tau} d\tau = -\frac{1}{9} e^{-2{,}25\tau} \Big|_0^t = -\frac{1}{9}\left(e^{-2{,}25t} - 1\right) =$$

$$\Rightarrow h_u(t) = \left[\frac{1}{9}\left(1 - e^{-2{,}25t}\right)\right] u(t)$$

e) $h_u(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau =$

$$= \int_0^t (-0{,}25 + 0{,}5\tau)\, d\tau = -0{,}25\tau \Big|_0^t + \frac{0{,}5\tau^2}{2}\Big|_0^t =$$

$$= -0{,}25t + 0{,}25t^2$$

$$\Rightarrow h_u(t) = (0{,}25t^2 - 0{,}25t)\, u(t)$$

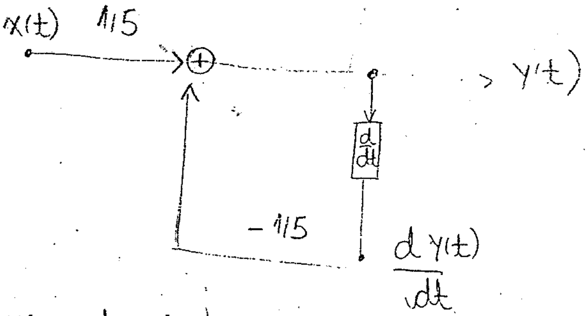
3) e 4)

a) $\frac{dy(t)}{dt} + 5y(t) = x(t)$

$$5y(t) = x(t) - \frac{dy(t)}{dt}$$

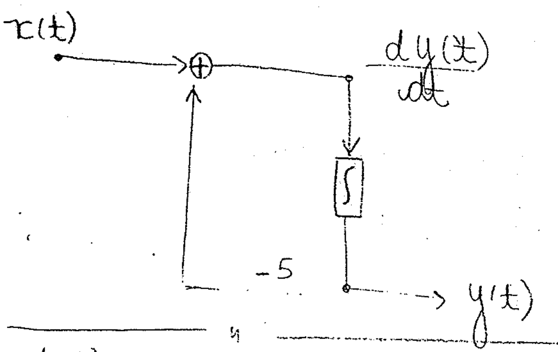
$$y(t) = \frac{x(t)}{5} - \frac{1}{5}\frac{dy(t)}{dt}$$

usando diferenciadores:



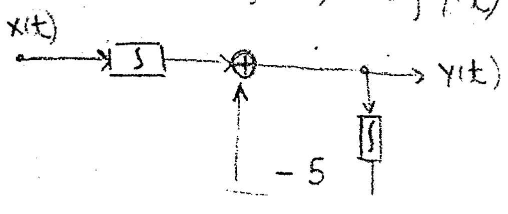
usando integradores:

$$\frac{dy(t)}{dt} = x(t) - 5y(t)$$



$\frac{dy(t)}{dt} + 5y(t) = x(t) \xrightarrow{\int} y(t) + 5\int y(t) = \int x(t)$

$$y(t) = \int x(t) - 5\int y(t)$$

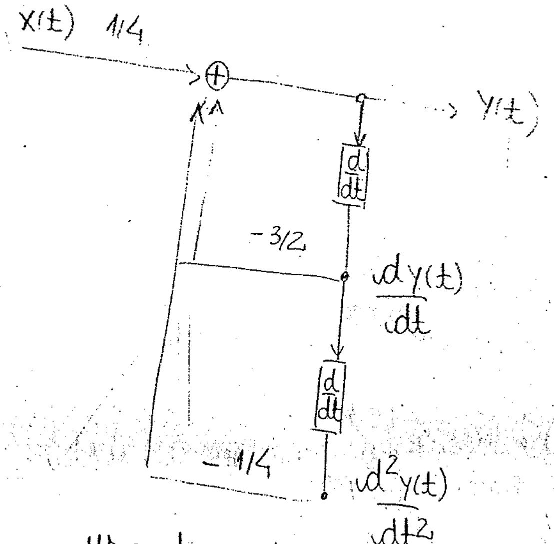


b) $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 4y(t) = x(t)$

usando diferenciadores:

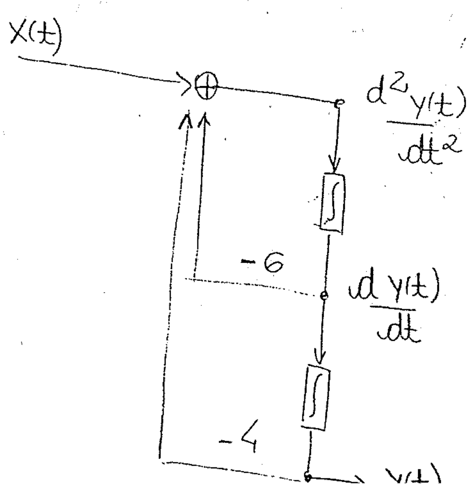
$$4y(t) = x(t) - 6\frac{dy(t)}{dt} - \frac{d^2y(t)}{dt^2}$$

$$y(t) = \frac{1}{4}x(t) - \frac{3}{2}\frac{dy(t)}{dt} - \frac{1}{4}\frac{d^2y(t)}{dt^2}$$



usando integradores:

$$\frac{d^2y(t)}{dt^2} = x(t) - 6\frac{dy(t)}{dt} - 4y(t)$$

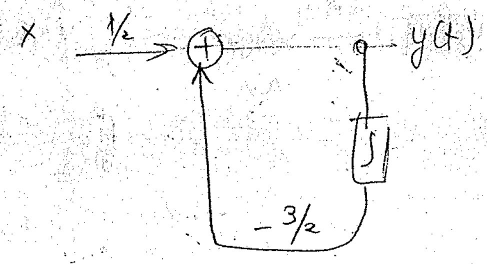


c) $2\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} \xrightarrow{\int} 2y(t) + 3\int y(t) = x(t)$

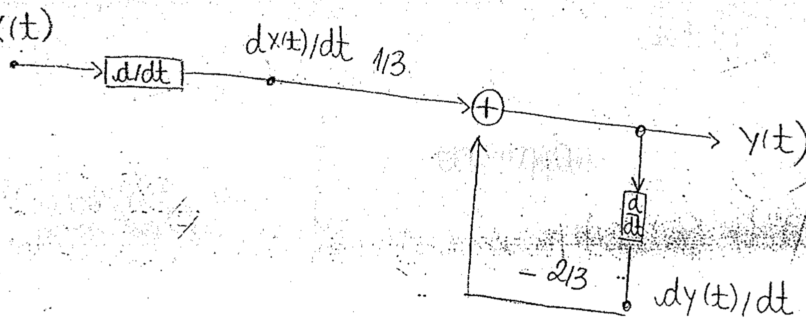
usando dif:

$$y = \frac{x(t)}{2} - \frac{3}{2}\int y(t)$$

$$3y(t) = \frac{dx(t)}{dt} - 2\frac{dy(t)}{dt}$$



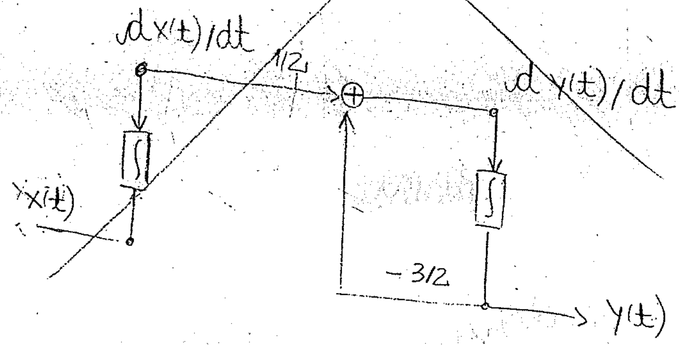
$$y(t) = \frac{1}{3}\frac{dx(t)}{dt} - \frac{2}{3}\frac{dy(t)}{dt}$$



usando int.:

$$2\frac{dy(t)}{dt} = \frac{dx(t)}{dt} - 3y(t)$$

$$\frac{dy(t)}{dt} = \frac{1}{2}\frac{dx(t)}{dt} - \frac{3}{2}y(t)$$

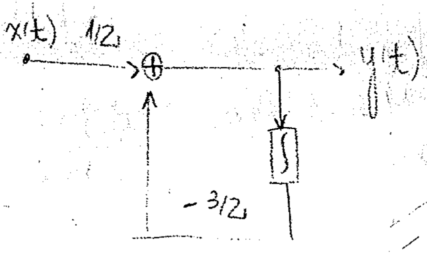


usando int.

$$2\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt}$$

$$\xrightarrow{\int} 2y(t) + 3\int y(t) = x(t)$$

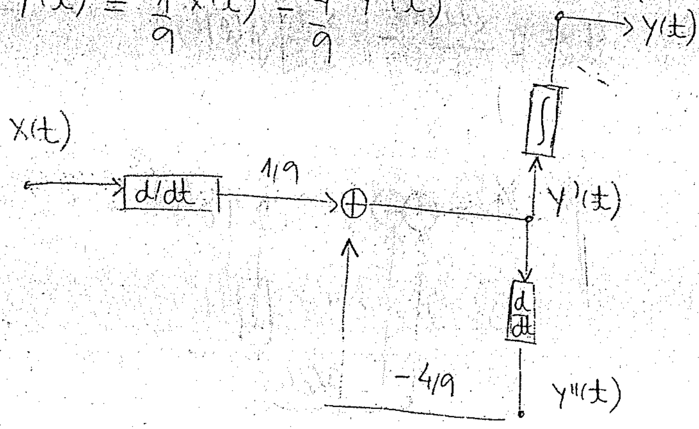
$$y(t) = \frac{x(t)}{2} - \frac{3}{2}\int y(t)$$



D) $4\,y''(t) + 9\,y'(t) = x'(t)$

usando dif.

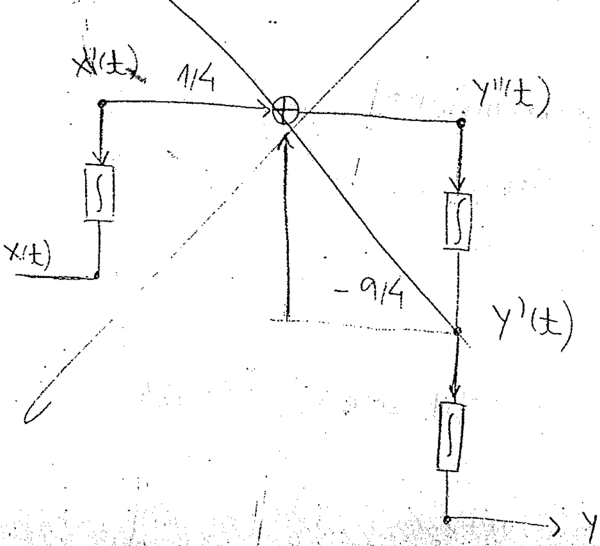
$$y'(t) = \frac{1}{9}\,x'(t) - \frac{4}{9}\,y''(t)$$



usando int.

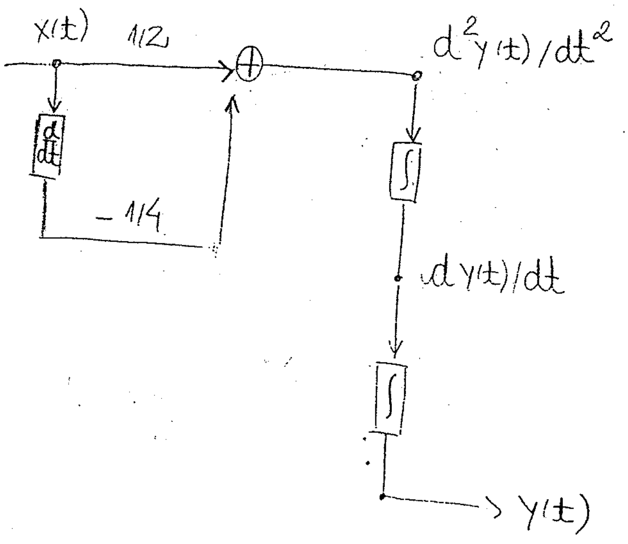
caderno

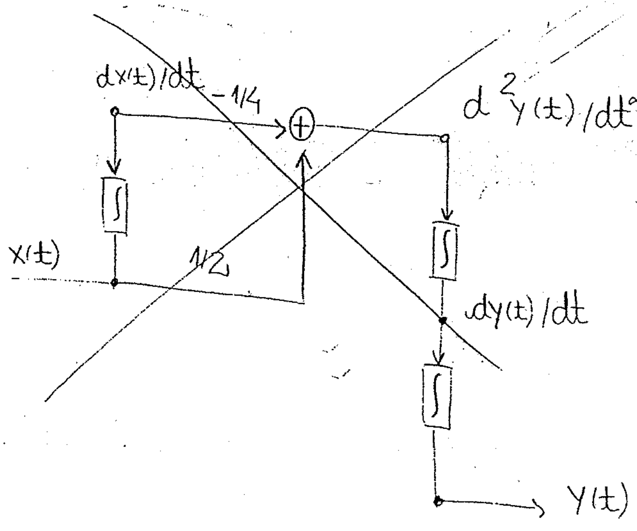
$$y''(t) = \frac{x'(t)}{4} - \frac{9}{4}\,y'(t)$$



e) $\dfrac{d^2y(t)}{dt^2} = \dfrac{1}{2}\,x(t) - \dfrac{1}{4}\,\dfrac{dx(t)}{dt}$

caderno

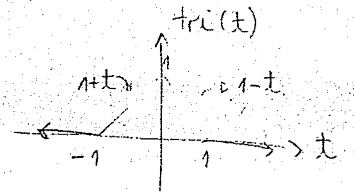




5) Propriedade da Amostragem do impulso $x(t) * A\delta(t-t_0) = A\,x(t-t_0)$

$x(t) = 2\,\text{tri}(t/4) * \delta(t-2) = 2\,\text{tri}\left(\frac{t-2}{4}\right)$

a) $x(1) = 2\,\text{tri}(-1/4) = 2.\left(1-\frac{1}{4}\right) = 2\cdot\frac{3}{4} = \frac{3}{2}$



b) $x(-1) = 2\,\text{tri}(-3/4) = 2.\left(1-\frac{3}{4}\right) = 2.\frac{1}{4} = 1/2$

c) $x(0) = 2\,\text{tri}(-1/2) = 2.(1-1/2) = 2.1/2 = 1$

D) $x(-\infty) = 2\,\text{tri}(-\infty) = 0$

6) $x(t) = \{e^{-(t-4)}u(t)\} * \delta(t-2)$

$x(t) = \{e^{-(t-2-4)}u(t-2)\} = \{e^{-(t-6)}u(t-2)\}$

a) $x(1) = 0 \Rightarrow u(t-2) = 0$

b) $x(-1) = 0 \Rightarrow u(t-2) = 0$

c) $x(0) = 0 \Rightarrow u(t-2) = 0$

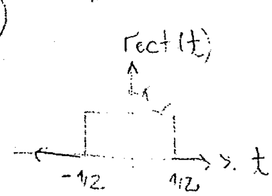
d) $x(-\infty) = 0 \Rightarrow u(t-2) = 0$

7) $x(t) = -5\,\text{rect}(t/2) * \{\delta(t+1) + \delta(t)\}$

Distributiva

$x(t) = -5\,\text{rect}(t/2) * \delta(t+1) - 5\,\text{rect}(t/2) * \delta(t)$

$x(t) = -5\,\text{rect}\left(\frac{t+1}{2}\right) - 5\,\text{rect}\left(t/2\right)$



a) $x(1/2) = -5\,\text{rect}(3/4) - 5\,\text{rect}(1/4)$

$x(1/2) = 0 - 5.1 = -5$

b) $x(-1/2) = -5\,\mathrm{rect}\left(1/4\right) - 5\,\mathrm{rect}\left(-1/4\right)$

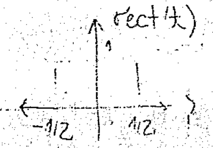
$x(-1/2) = -5 - 5 = -10$

c) $x(-5/2) = -5\,\mathrm{rect}\left(-3/4\right) - 5\,\mathrm{rect}\left(-5/4\right)$

$x(-5/2) = 0 - 0 = 0$

8)

a) $x(t) = \text{rect}(t) = u(t+1/2) - u(t-1/2)$



$h(t) = \text{rect}(t) = u(t+1/2) - u(t-1/2)$

$y(t) = x(t) * h(t)$

$y'(t) = x(t) * h'(t)$

$y'(t) = x(t) * \{\delta(t+1/2) - \delta(t-1/2)\}$

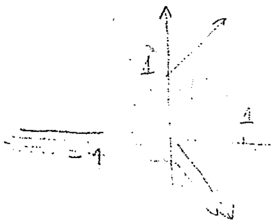
distributiva

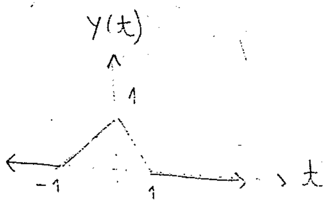
$y'(t) = \{u(t+1/2) - u(t-1/2)\} * \{\delta(t+1/2) - \delta(t-1/2)\}$

$y'(t) = u(t+1/2) * \delta(t+1/2) - u(t+1/2) * \delta(t-1/2) - u(t-1/2) * \delta(t+1/2)$
$+ u(t-1/2) * \delta(t-1/2)$ (Amostragem do impulso)

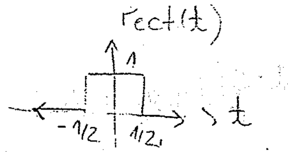
$y'(t) = u(t+1) - u(t) - u(t) + u(t-1)$

$\Rightarrow y(t) = \text{rampa}(t+1) - 2\,\text{rampa}(t) + \text{rampa}(t-1)$

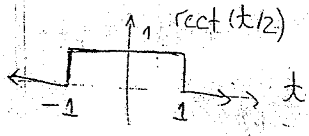




b) $x(t) = \text{rect}(t) = u(t+1/2) - u(t-1/2)$



$h(t) = \text{rect}(t/2) = u(t+1) - u(t-1)$



$y(t) = x(t) * h(t)$

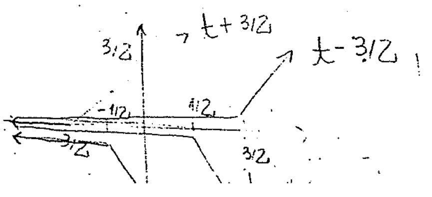
$y'(t) = x(t) * h'(t)$

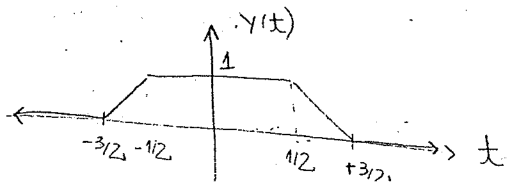
$y'(t) = \{u(t+1/2) - u(t-1/2)\} * \{\delta(t+1) - \delta(t-1)\}$

$y'(t) = u(t+1/2) * \delta(t+1) - u(t+1/2) * \delta(t-1) - u(t-1/2) * \delta(t+1) +$
$+ u(t-1/2) * \delta(t-1)$

$y'(t) = u(t+3/2) - u(t-1/2) - u(t+1/2) + u(t-3/2)$

$\Rightarrow y(t) = \text{rampa}(t+3/2) - \text{rampa}(t-1/2) - \text{rampa}(t+1/2) + \text{rampa}(t-3/2)$

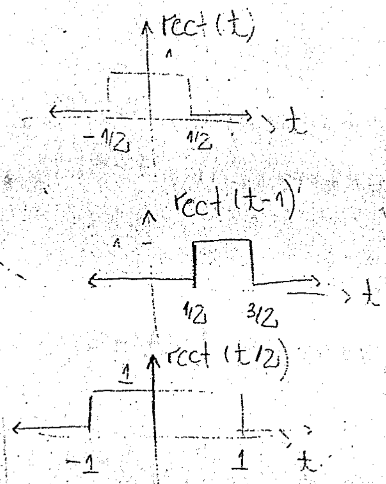




c)

$$x(t) = \text{rect}(t-1) = u(t-1/2) - u(t-3/2)$$

$$h(t) = \text{rect}(t/2) = u(t+1) - u(t-1)$$



$$y(t) = x(t) * h(t)$$

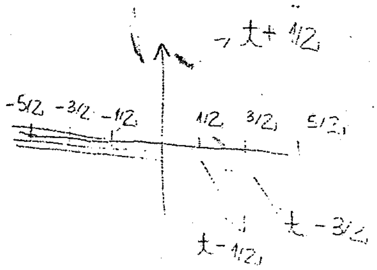
$$y'(t) = x(t) * h'(t)$$

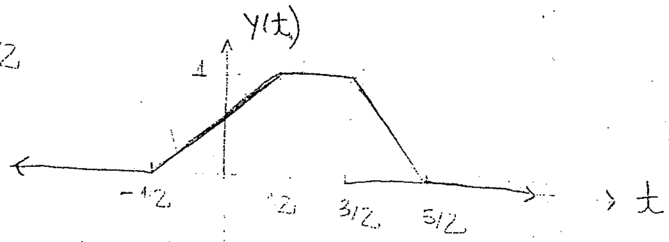
$$y'(t) = \{u(t-1/2) - u(t-3/2)\} * \{\delta(t+1) - \delta(t-1)\}$$

$$y'(t) = u(t-1/2) * \delta(t+1) - u(t-1/2) * \delta(t-1) - u(t-3/2) * \delta(t+1) + u(t-3/2) * \delta(t-1)$$

$$y'(t) = u(t+1/2) - u(t-3/2) - u(t-1/2) + u(t-5/2)$$

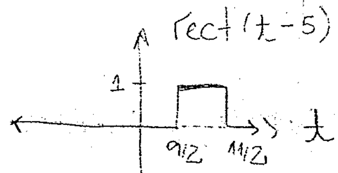
$$\Rightarrow y(t) = \text{rampa}(t+1/2) - \text{rampa}(t-3/2) - \text{rampa}(t-1/2) + \text{rampa}(t-5/2)$$





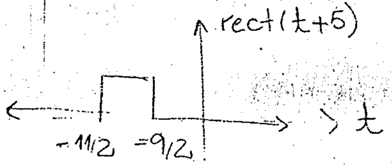
D) $x(t) = \text{rect}(t-5) + \text{rect}(t+5)$

$h(t) = \text{rect}(t-4) + \text{rect}(t-4) = 2\,\text{rect}(t-4)$



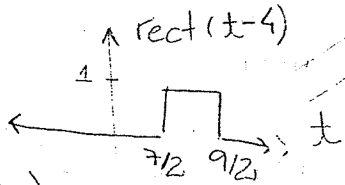
$t - 4 = -1/2$

$t = 4 - 1/2 = \frac{8}{2} - \frac{1}{2} = 7/2$



$t - 4 = 1/2$

$t = 8/2 + 1/2 = 9/2$



$$x(t) = \underbrace{u(t-9/2) - u(t-11/2)}_{\text{rect}(t-5)} + \underbrace{u(t+11/2) - u(t+9/2)}_{\text{rect}(t+5)}$$

$$h(t) = 2\left[\underbrace{u(t-7/2) - u(t-9/2)}_{\text{rect}(t-4)}\right]$$

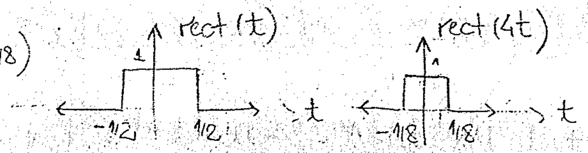
$$y(t) = x(t) * h(t)$$

$$y(t) = [\text{rect}(t-5) + \text{rect}(t+5)] * 2\,\text{rect}(t-4)$$

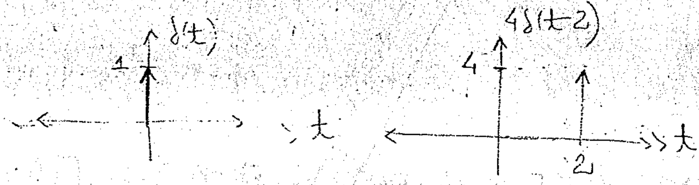
$$y(t) = 2\left[\text{rect}(t-5) * \text{rect}(t-4) + \text{rect}(t+5) * \text{rect}(t-4)\right]$$

e) $x(t) = \mathrm{rect}(4t) = u(t+1/8) - u(t-1/8)$

$h(t) = 4\delta(t-2) = 4\delta(t-2)$



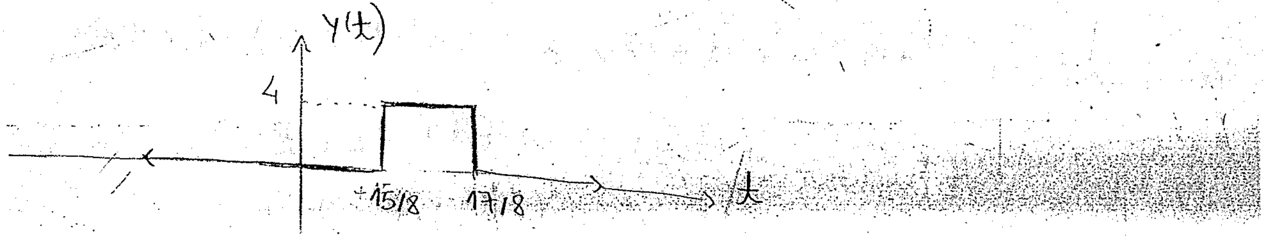
$y(t) = x(t) * h(t)$



$$y(t) = [u(t+1/8) - u(t-1/8)] * [4\delta(t-2)]$$

$$y(t) = 4[u(t+1/8) * \delta(t-2) - u(t-1/8) * \delta(t-2)]$$

$$\Rightarrow y(t) = 4[u(t-15/8) - u(t-17/8)]$$

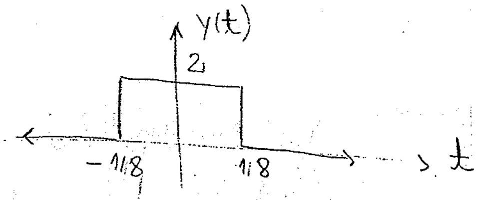


f) $x(t) = \mathrm{rect}(4t) = u(t+1/8) - u(t-1/8)$

$h(t) = 4\delta(2t) = \frac{4}{2}\delta(t) = 2\delta(t)$

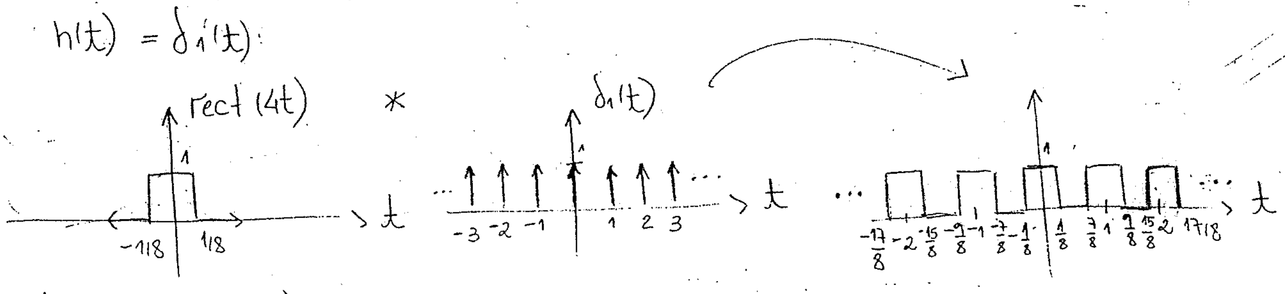
$$y(t) = x(t) * h(t) = 2[u(t+1/8) * \delta(t) - u(t-1/8) * \delta(t)]$$

$$\Rightarrow y(t) = 2[u(t+1/8) - u(t-1/8)]$$

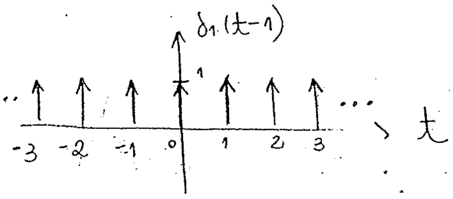


g) $x(t) = \mathrm{rect}(4t)$

$h(t) = \delta_1(t)$



h) $h(t) = \delta_1(t-1)$



$\Rightarrow$ mesmo caso que o anterior!

$$y(t) = 2\left\{ \left[u(t-9/2) - u(t-11/2)\right] * \left[u(t-7/2) - u(t-9/2)\right] + \right.$$

$$\left. + \left[u(t+11/2) - u(t+9/2)\right] * \left[u(t-7/2) - u(t-9/2)\right] \right\}$$

$$y_1(t) = 2\left\{ \left[u(t-9/2) - u(t-11/2)\right] * \left[u(t-7/2) - u(t-9/2)\right] \right\}$$

$$y_1'(t) = 2\left\{ \left[u(t-9/2) - u(t-11/2)\right] * \left[\delta(t-7/2) - \delta(t-9/2)\right] \right\}$$

$$\Rightarrow y_1'(t) = 2\left\{ u(t-8) - u(t-9) - u(t-9) + u(t-10) \right\}$$

$$y_2(t) = 2\left\{ \left[u(t+11/2) - u(t+9/2)\right] * \left[u(t-7/2) - u(t-9/2)\right] \right\}$$

$$y_2'(t) = 2\left\{ \left[u(t+11/2) - u(t+9/2)\right] * \left[\delta(t-7/2) - \delta(t-9/2)\right] \right\}$$

$$y_2'(t) = 2\left\{ u(t+2) - u(t+1) - u(t+1) + u(t) \right\}$$

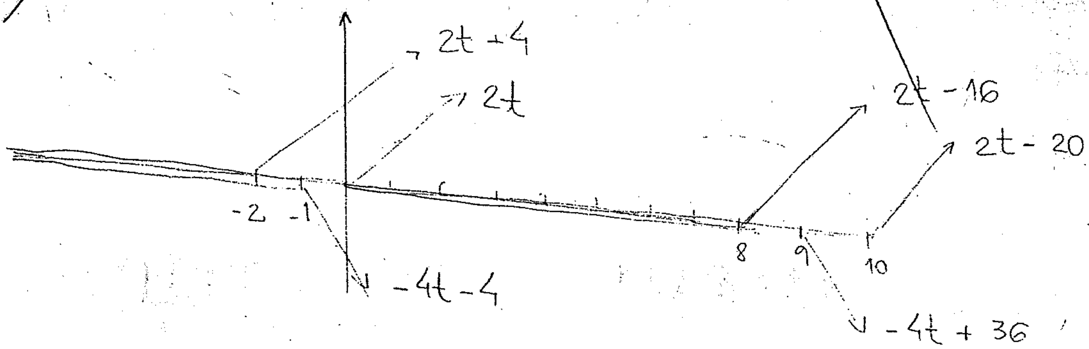
$$\Rightarrow y_2'(t) = 2\left\{ u(t+2) - 2u(t+1) + u(t) \right\}$$

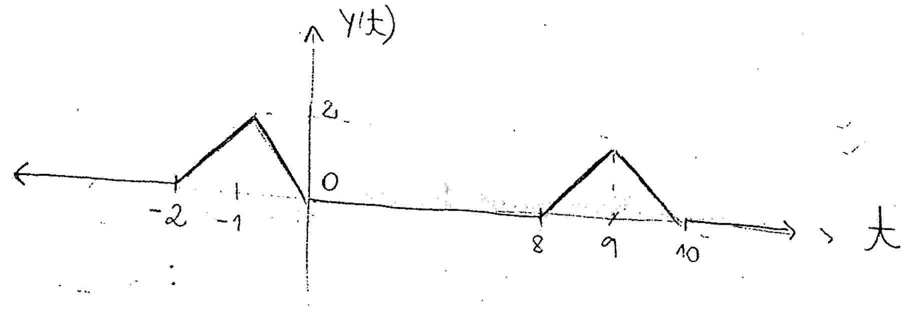
$$y(t) = 2\left\{ \text{rampa}(t-8) \underbrace{- \text{rampa}(t-9) - \text{rampa}(t-9)}_{-2\,\text{rampa}(t-9)} + \text{rampa}(t-10) \right\}$$

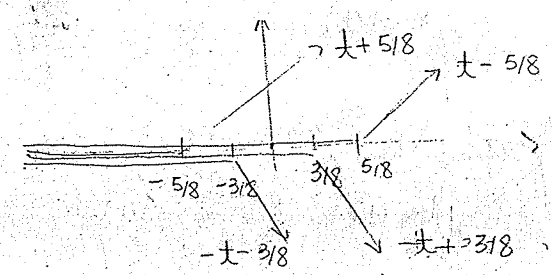
$$+ \; 2\left\{ \text{rampa}(t+2) - 2\,\text{rampa}(t+1) + \text{rampa}(t) \right\}$$

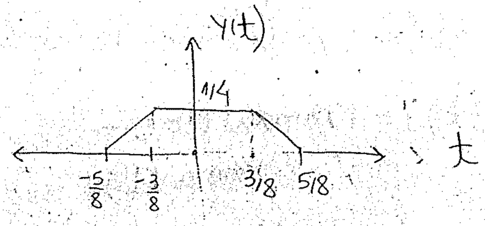
$$\Rightarrow y(t) = 2\,\text{rampa}(t-8) - 4\,\text{rampa}(t-9) + 2\,\text{rampa}(t-10)$$

$$+ \; 2\,\text{rampa}(t+2) - 4\,\text{rampa}(t+1) + 2\,\text{rampa}(t)$$





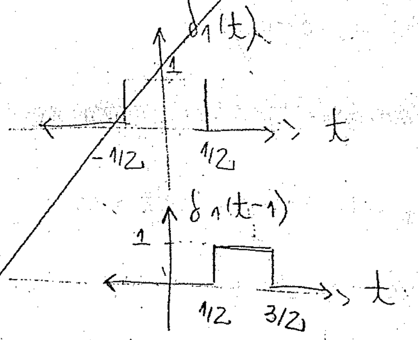




errado (mesmo de g)

h) $x(t) = rect(4t) = u(t+1/8) - u(t-1/8)$

$h(t) = \delta_1(t-1) = u(t-1/2) - u(t-3/2)$



$y(t) = x(t) * h(t)$

$y'(t) = [u(t+1/8) - u(t-1/8)] *$

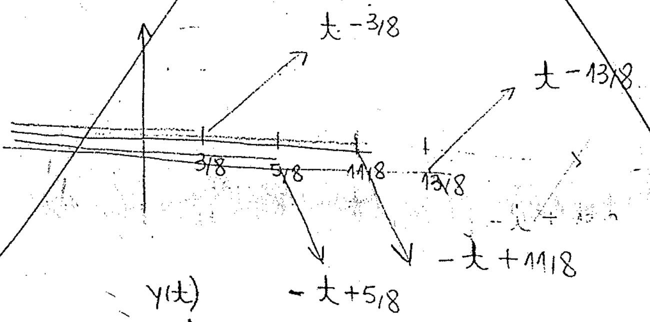
$[\delta(t-1/2) - \delta(t-3/2)]$

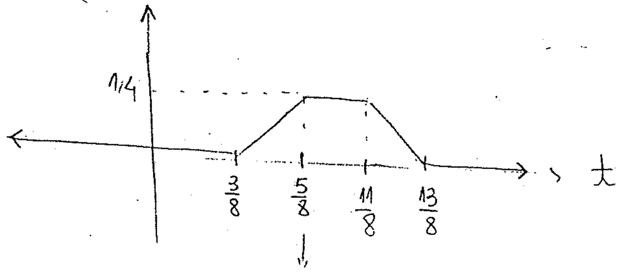
$y'(t) = u(t+1/8) * \delta(t-1/2) - u(t+1/8) * \delta(t-3/2)$

$- u(t-1/8) * \delta(t-1/2) + u(t-1/8) * \delta(t-3/2)$

$y'(t) = u(t-3/8) - u(t-11/8) - u(t-5/8) + u(t-13/8)$

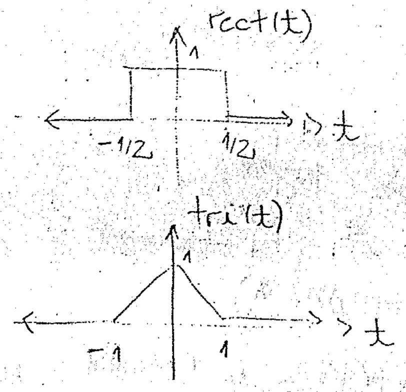
$\Rightarrow y(t) = +rampa(t-3/8) - rampa(t-11/8) - rampa(t-5/8) + rampa(t-13/8)$





i) $x(t) = \mathrm{rect}(t) = u(t+1/2) - u(t-1/2)$

$h(t) = \mathrm{tri}(t) = \mathrm{rampa}(t+1) - 2\,\mathrm{rampa}(t) + \mathrm{rampa}(t-1)$



$y(t) = x(t) * h(t)$

$y'(t) = x'(t) * h(t)$

$y'(t) = h(t) * x'(t)$

$y'(t) = [\mathrm{rampa}(t+1) - 2\,\mathrm{rampa}(t) + \mathrm{rampa}(t-1)] * [\delta(t+1/2) - \delta(t-1/2)]$

$y'(t) = \mathrm{rampa}(t+1) * \delta(t+1/2) - \mathrm{rampa}(t+1) * \delta(t-1/2)$

$\quad - 2\,\mathrm{rampa}(t) * \delta(t+1/2) + 2\,\mathrm{rampa}(t) * \delta(t-1/2)$

$\quad + \mathrm{rampa}(t-1) * \delta(t+1/2) - \mathrm{rampa}(t-1) * \delta(t-1/2)$

$y'(t) = \mathrm{rampa}(t+3/2) - \mathrm{rampa}(t+1/2) -$

$\quad - 2\,\mathrm{rampa}(t+1/2) + 2\,\mathrm{rampa}(t-1/2)$

$\quad + \mathrm{rampa}(t-1/2) - \mathrm{rampa}(t-3/2)$

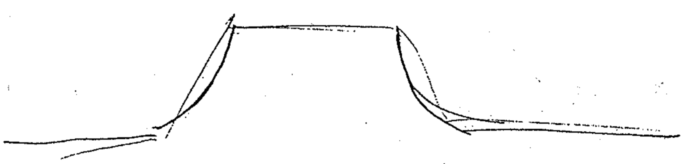
$y'(t) = \mathrm{rampa}(t+3/2) - \mathrm{rampa}(t-3/2) - 3\,\mathrm{rampa}(t+1/2)$

$\quad + 3\,\mathrm{rampa}(t-1/2)$

$\Rightarrow \mathrm{rampa}(t) = t\,u(t)$

$$\Rightarrow y(t) = \frac{(t+3/2)^2}{2} u(t+3/2) - \frac{(t-3/2)^2}{2} u(t-3/2)$$

$$- 3\,\frac{(t+1/2)^2}{2} u(t+1/2) + 3\,\frac{(t-1/2)^2}{2} u(t-1/2)$$



j) $x(t) = e^{-t} u(t)$

$h(t) = e^{-t} u(t)$

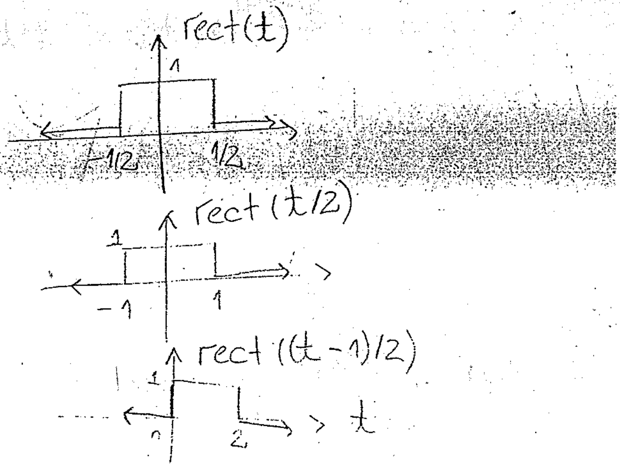
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, e^{-(t-\tau)} u(t-\tau)\, d\tau$$

$$y(t) = \int_{0}^{t} e^{-\tau} \cdot e^{-t} \cdot e^{\tau}\, d\tau = e^{-t} \cdot \tau \Big|_0^t = t\, e^{-t} u(t)$$

k) $x(t) = \mathrm{rect}\left((t-1)/2\right) = u(t) - u(t-2)$

$h(t) = 4 e^{-4t} u(t)$



$y(t) = x(t) * h(t)$

$y'(t) = x'(t) * h(t)$

$y'(t) = h(t) * x'(t)$

$y'(t) = 4 e^{-4t} u(t) * [\delta(t) - \delta(t-2)]$

$y'(t) = 4 e^{-4t} u(t) - 4 e^{-4(t-2)} u(t-2)$

$$\int_{-\infty}^{t}$$

$$y(t) = \int_0^t 4 e^{-4\tau}\, d\tau - \int_2^t 4 e^{-4(\tau-2)}\, d\tau$$

$$y(t) = 4 \cdot -\frac{1}{4} e^{-4\tau} \Big|_0^t - 4 \cdot -\frac{1}{4} e^{-4(\tau-2)} \Big|_2^t$$

$$y(t) = -\left(e^{-4t} - 1\right) + \left(e^{-4(t-2)} - 1\right) \Longrightarrow \boxed{y(t) = -\left(e^{-4t} - 1\right) u(t) + \left(e^{-4(t-2)} - 1\right) u(t-2)}$$

$$\cancel{y(t) = 1 - e^{-4t} - 1 + e^{-4(t-2)} = e^{-4(t-2)} - e^{-4t}}$$

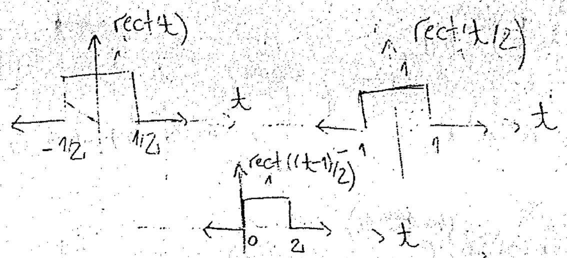
$u(t)$

$$\boxed{-\left(e^{-4t} - 1\right) u(t) + \left(e^{-4(t-2)} - 1\right) u(t-2)}$$

l)

$x(t) = \mathrm{rect}((t-1)/2) = u(t) - u(t-2)$

$h(t) = \delta(t) - 4e^{-4t}u(t)$



$y(t) = x(t) * h(t)$

$y'(t) = x'(t) * h(t)$

$y'(t) = [\delta(t) - 4e^{-4t}u(t)] * [\delta(t) - \delta(t-2)]$

$y'(t) = \delta(t) * \delta(t) - \delta(t) * \delta(t-2) - 4e^{-4t}u(t) * \delta(t)$
$+ 4e^{-4t}u(t) * \delta(t-2)$

$\boxed{\int_{-\infty}^{t}}$

$y'(t) = \delta(t) - \delta(t-2) - 4e^{-4t}u(t) + 4e^{-4(t-2)}u(t-2)$

$y(t) = u(t) - u(t-2) - 4\int_0^t e^{-4\tau}d\tau + 4\int_2^t e^{-4(\tau-2)}d\tau$

$y(t) = u(t) - u(t-2) - \cancel{4}\cdot -\frac{1}{\cancel{4}} e^{-4\tau}\Big|_0^t + \cancel{4}\cdot -\frac{1}{\cancel{4}} e^{-4(\tau-2)}\Big|_2^t$

$y(t) = u(t) - u(t-2) + (e^{-4t} - 1)u(t) - (e^{-4(t-2)} - 1)u(t-2)$

m) $x(t) = \mathrm{rect}(t/2)\,\mathrm{sinc}(t)$

$h(t) = \delta_2(t) = \frac{1}{2}u(t+1) - \frac{1}{2}u(t-1)$

$y'(t) = x(t) * h'(t)$

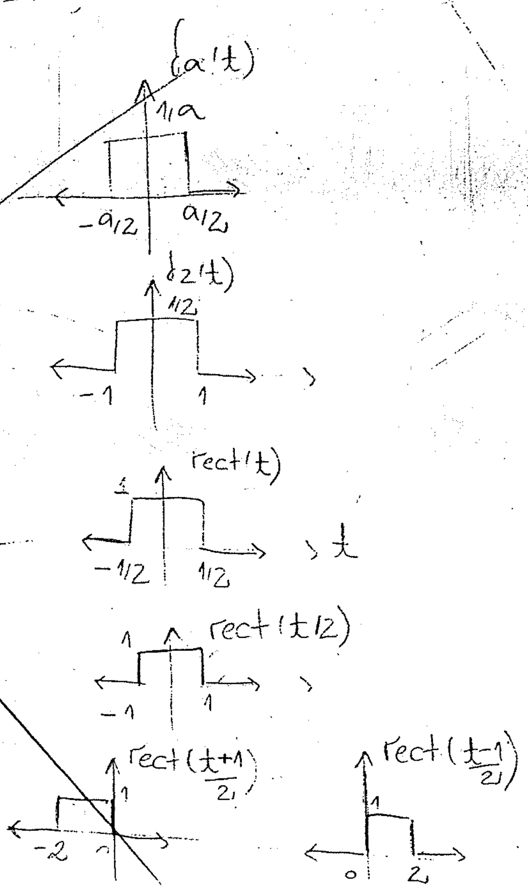
$y'(t) = \mathrm{rect}(t/2)\,\mathrm{sinc}(t) * [\frac{1}{2}\delta(t+1) - \frac{1}{2}\delta(t-1)]$

$y'(t) = \mathrm{rect}(t/2)\,\mathrm{sinc}(t) * \frac{1}{2}\delta(t+1) -$
$- \mathrm{rect}(t/2)\,\mathrm{sinc}(t) * \frac{1}{2}\delta(t-1)$

$y'(t) = \frac{1}{2}\mathrm{rect}\left(\frac{t+1}{2}\right)\mathrm{sinc}(t+1) - \frac{1}{2}\mathrm{rect}\left(\frac{t-1}{2}\right)\mathrm{sinc}(t-1)$

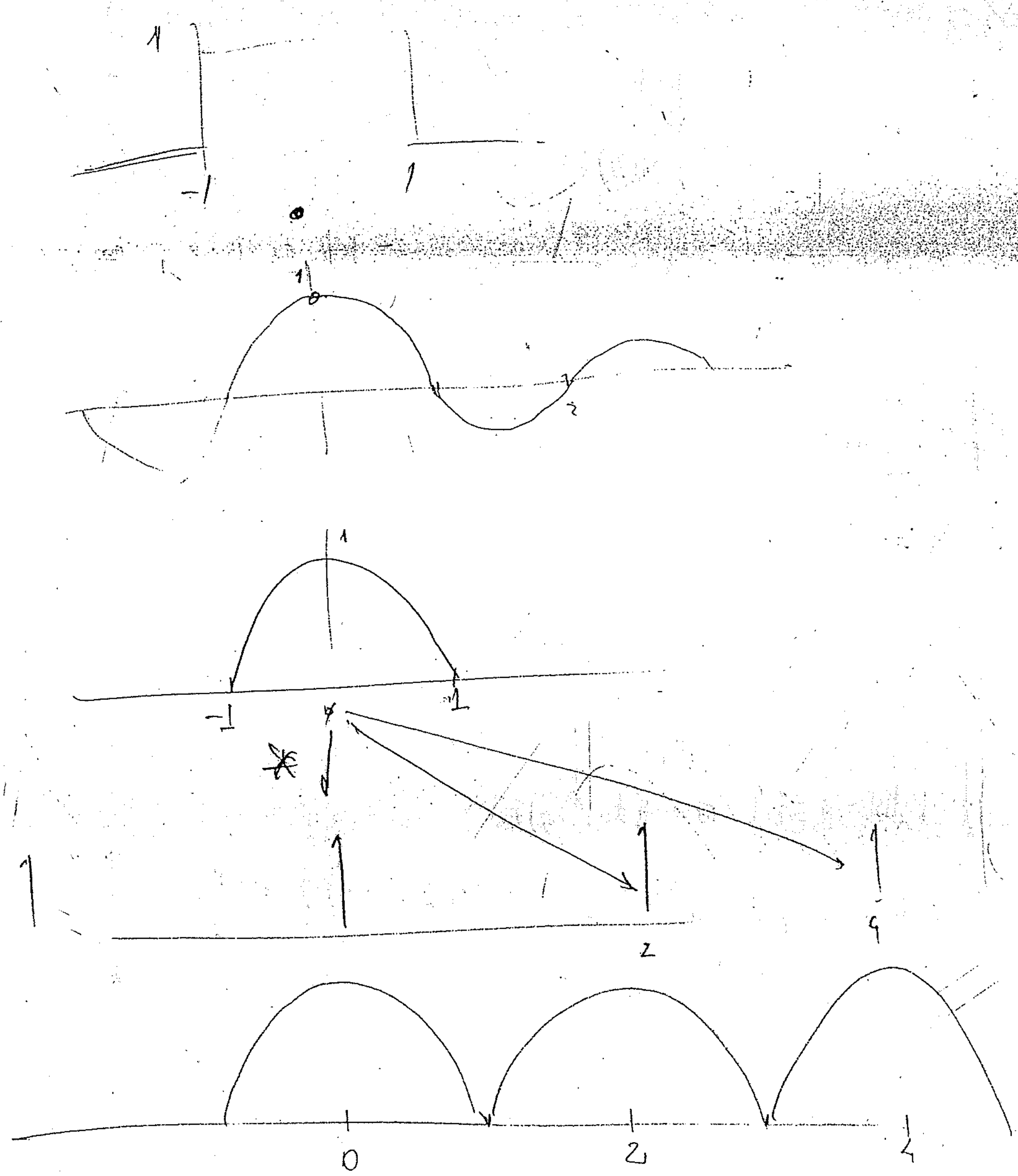
$y(t) = \frac{1}{2}\int_{-2}^{0}\mathrm{sinc}(\tau+1)\,d\tau - \frac{1}{2}\int_0^2 \mathrm{sinc}(\tau-1)\,d\tau$

$y(t) =$



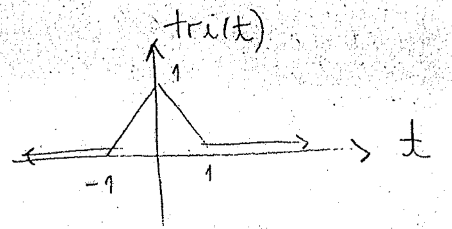
m)

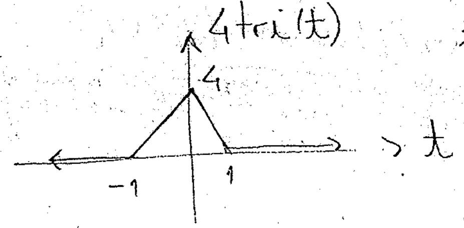
$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



a)

b) $x(t) = 4\,tri(t) * \delta_4(t)$





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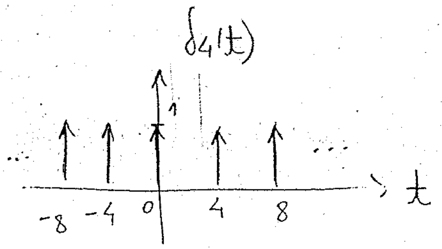
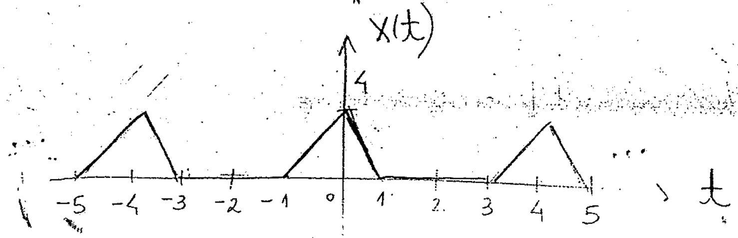


$\Downarrow$



$\Longrightarrow$ $\boxed{E_x = \infty}$

$$P_x = \frac{1}{T}\int_T |x(t)|^2 dt$$

$$P_x = 2\int_0^1 (-4(t-1))^2 dt$$

$y - y_0 = m(x - x_0)$

$0 - 4 = m(1-0)$

$m = -4$

$y - 4 = -4(x)$

$y = -4x + 4$

$y = -4(x-1)$

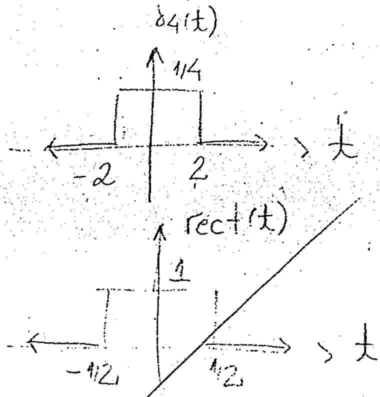
4

$$P_x = \frac{1}{2}\int_0^1 16(t-1)^2 dt$$

$$P_x = 8\,\frac{(t-1)^3}{3}\Bigg|_0^1 = \frac{8}{3}\left(0-(-1)\right) = \frac{8}{3}$$

9)

a) $x(t) = 4\,\text{rect}(t) * \delta_4(t)$



$x(t) = 4\,[u(t+1/2) - u(t-1/2)] *$

$\left[\frac{1}{4}u(t+2) - \frac{1}{4}u(t-2)\right]$

$x(t) = \cancel{4}\cdot\frac{1}{\cancel{4}}\,[u(t+1/2) - u(t-1/2)] *$

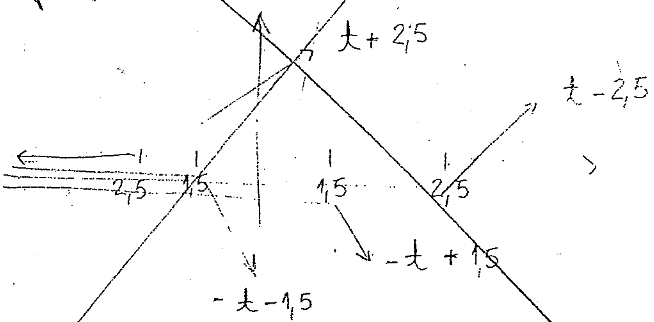
$[u(t+2) - u(t-2)]$

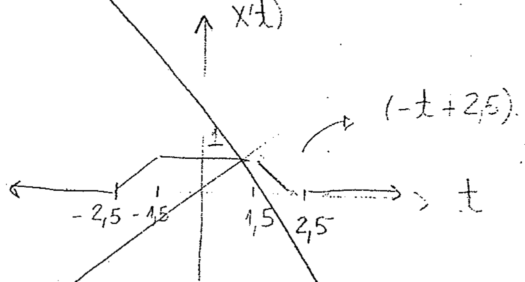
$x'(t) = [u(t+2) - u(t-2)] * [\delta(t+1/2) - \delta(t-1/2)]$

$x'(t) = u(t+2{,}5) - u(t+1{,}5) - u(t-1{,}5) + u(t-2{,}5)$

$x(t) = \text{rampa}(t+2{,}5) - \text{rampa}(t+1{,}5) - \text{rampa}(t-1{,}5) +$

$+\ \text{rampa}(t-2{,}5)$





P → não é periódico

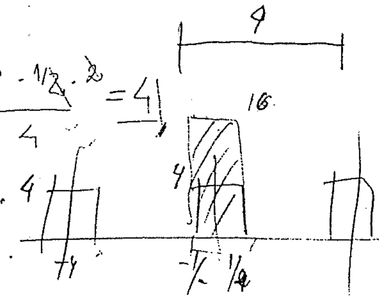
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt = 2\cdot\int_0^{1,5} 1\,dt + 2\int_{1,5}^{2,5} (-t+2{,}5)^2\,dt$$

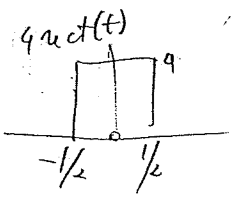
$$E_x = 3 - 2\,\frac{(-t+2{,}5)^3}{3}\Bigg|_{1,5}^{2,5}$$

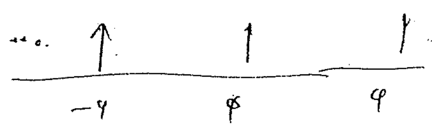
$$E_x = 3 - 2\,(0 - 1/3)$$

$$E_x = 3 + \frac{2}{3} = 11/3$$

$$\frac{2\cdot\int_0^{1/2} 16\,dt}{4} = \frac{16\cdot 1/2\cdot 2}{4} = 4$$







Area = 16

$P_x = 16 = 0$

2ª lista - Exercícios de série de Fourier

1) $g(t) = (1+j)e^{j4\pi t} + (1-j)e^{-j4\pi t} =$

$= (1+j)(\cos 4\pi t + j\,\text{sen} 4\pi t) + (1-j)(\cos 4\pi t - j\,\text{sen} 4\pi t) =$

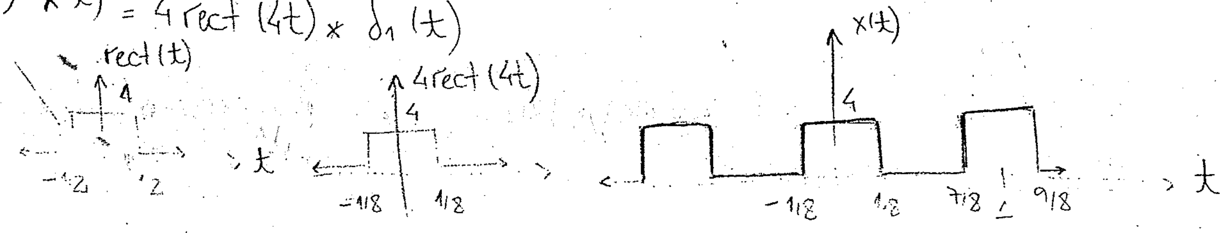
$= \cos 4\pi t + j\,\text{sen} 4\pi t + \cancel{j\cos 4\pi t} - \text{sen} 4\pi t + \cos 4\pi t - \cancel{j\,\text{sen} 4\pi t}$
$\cancel{- j\cos 4\pi t} - \text{sen} 4\pi t =$

$= 2\cos 4\pi t - 2\,\text{sen} 4\pi t$

$g(t) = 2[\cos(4\pi t) - \text{sen}(4\pi t)]$ //

2)

a) $x(t) = 4\,\text{rect}(4t) * \delta_1(t)$



$X[K] = \frac{1}{T_0}\int_T x(t)e^{-jK\Omega t}dt = \frac{1}{T_0}\int_T x(t)e^{-jK\Omega_0 t}dt =$ $\quad T_0 = 1 \rightarrow \Omega_0 = \frac{2\pi}{T_0} = 2\pi$

$X[K] = \frac{1}{1}\int_{-1/8}^{1/8} 4e^{-jK2\pi t}dt = \frac{4}{-jK2\pi}\left[e^{-jK2\pi t}\right]_{-1/8}^{1/8} =$

$= \frac{4}{-jK2\pi}\left[e^{-jK2\pi/8} - e^{+jK2\pi/8}\right] = \frac{4}{jK2\pi}\left[e^{jK\pi/4} - e^{-jK\pi/4}\right] =$

$\text{sinc}\,x = \frac{\text{sen}\,\pi x}{\pi x}$ $\quad = \frac{4}{K\pi}\left[\frac{e^{jK\pi/4} - e^{-jK\pi/4}}{2j}\right] = \boxed{\frac{4}{K\pi}\text{sen}(K\pi/4)}$ //

ou $\frac{4}{K\pi}\text{sen}(K\pi/4) = \frac{4}{4\frac{K\pi}{4}}\text{sen}(K\pi/4) = \frac{\text{sen}(K\pi/4)}{K\pi/4} = \boxed{\text{sinc}\left(\frac{K}{4}\right)}$ //

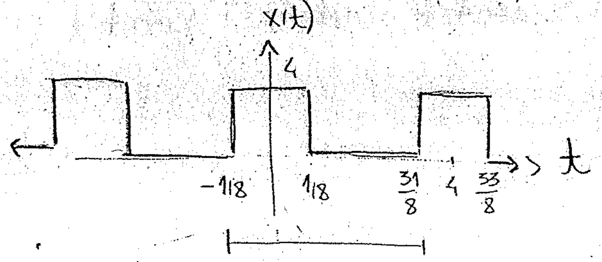
$SF\left\{\frac{1}{1/4}\text{rect}\left(\frac{t}{1/4}\right) * \delta_1(t)\right\} = \frac{2\pi}{2\pi}\text{sinc}\left(\frac{1}{4}\cdot K\cdot\frac{2\pi}{2\pi}\right) = \text{sinc}(K/4)$

$\boxed{X[K] = \text{sinc}(K/4)}$ //

b) $x(t) = 4\,\mathrm{rect}(4t) * \delta_4(t) \qquad T_0 = 4,\ \Omega_0 = \frac{2\pi}{4} = \pi/2$

$$SF\left\{ \frac{1}{1/4}\,\mathrm{rect}\left(\frac{t}{1/4}\right) * \delta_4(t) \right\} = \frac{\pi/2}{2\pi}\,\mathrm{sinc}\left(\frac{1}{4}\cdot k \cdot \frac{\pi/2}{2\pi}\right) =$$

$$= \frac{1}{4}\,\mathrm{sinc}\left(\frac{k}{16}\right) \rightarrow \boxed{X[k] = \frac{1}{4}\,\mathrm{sinc}\left(k/16\right)}$$



$T_0 = 4$

$\Omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$X[k] = \frac{1}{T_0}\int_T x(t)\,e^{-jk\Omega t}\,dt = \frac{1}{4}\int_{-1/8}^{1/8} 4\,e^{-jk\pi/2\,t}\,dt =$$

$$= \frac{1}{-jk\pi/2}\left[e^{-jk\pi/2\,t}\right]_{-1/8}^{1/8} = \frac{1}{-jk\pi/2}\left[e^{-jk\pi/16} - e^{+jk\pi/16}\right] =$$

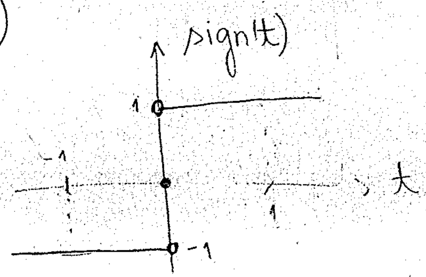
$$= \frac{1}{\frac{4jk\pi/2}{4}}\left[e^{jk\pi/16} - e^{-jk\pi/16}\right] = \frac{4}{2jk\pi}\left[e^{jk\pi/16} - e^{-jk\pi/16}\right]$$

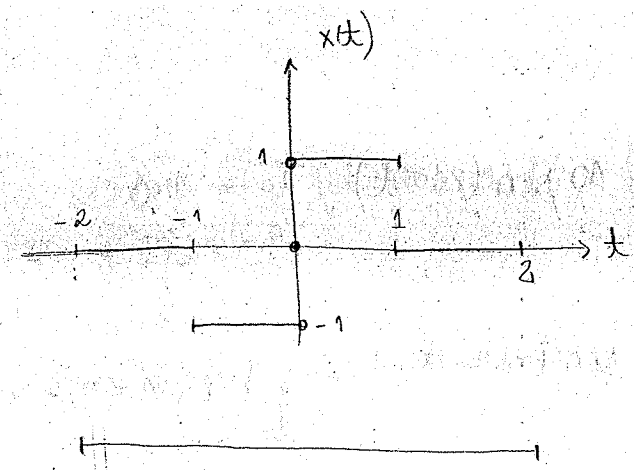
$$= \frac{4}{k\pi}\left[\frac{e^{jk\pi/16} - e^{-jk\pi/16}}{2j}\right] = \frac{4}{16\,\frac{k\pi}{16}}\,\mathrm{sen}\left(\frac{k\pi}{16}\right) =$$

$$= \frac{1}{4}\,\frac{\mathrm{sen}\left(k\pi/16\right)}{k\pi/16} = \frac{1}{4}\,\mathrm{sinc}\left(k/16\right)$$

$$\boxed{X[k] = \frac{1}{4}\,\mathrm{sinc}\left(k/16\right)}$$

ic)





$T_0 = 4$

$\Omega_0 = \frac{2\pi}{4} = \pi/2$

$$X[k] = \frac{1}{4}\int_{-1}^{1} x(t)\, e^{-jk\pi/2\, t}\, dt =$$

$$= \frac{1}{4}\left[\int_{-1}^{0} -e^{-jk\pi/2\, t}\, dt + \int_{0}^{1} e^{-jk\pi/2\, t}\, dt\right] =$$

$$= \frac{1}{4}\left[\frac{1}{jk\pi/2}\, e^{-jk\pi/2\, t}\Big|_{-1}^{0} + \frac{1}{-jk\pi/2}\, e^{-jk\pi/2\, t}\Big|_{0}^{1}\right] =$$

$$= \frac{1}{4}\left[\frac{1}{jk\pi/2}\left(1 - e^{jk\pi/2}\right) - \frac{1}{jk\pi/2}\left(e^{-jk\pi/2} - 1\right)\right] =$$

$$= \frac{1}{4}\left[\frac{1}{jk\pi/2} - \frac{e^{jk\pi/2}}{jk\pi/2} - \frac{e^{-jk\pi/2}}{jk\pi/2} + \frac{1}{jk\pi/2}\right] =$$

$$= \frac{1}{4}\left[\frac{2}{jk\pi/2} - \frac{\left(e^{jk\pi/2} + e^{-jk\pi/2}\right)}{jk\pi/2}\right] = \frac{1}{4j\frac{k\pi}{2}}\left[2 - \left(e^{jk\pi/2} + e^{-jk\pi/2}\right)\right]$$

$$= \frac{1}{jk2\pi}\left[2 - \left(e^{jk\pi/2} + e^{-jk\pi/2}\right)\right] = \frac{1}{jk\pi}\left[\frac{2}{2} - \frac{\left(e^{+jk\pi/2} + e^{-jk\pi/2}\right)}{2}\right]$$

$$= \frac{1}{jk\pi}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right]$$

$$\boxed{X[k] = \frac{1}{jk\pi}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right]}$$

3)

a) $x(t) = 10\,\text{sen}(20\pi t)$; $T_0 = 1/10$

$\Omega_0 = \dfrac{2\pi}{\frac{1}{10}} = 20\pi$

Como:

$$SF\{\text{sen}(n\Omega_0 t)\} = \frac{j}{2}\left[\delta(k+n) - \delta(k-n)\right]$$

$$X[k] = 10\,\frac{j}{2}\left[\delta(k+1) - \delta(k-1)\right] = \boxed{5j\left[\delta(k+1) - \delta(k-1)\right]}$$

b) $x(t) = 2\cos(100\pi(t-0{,}005))$, $T_0 = 1/50$

$\Omega_0 = \dfrac{2\pi}{1/50} = 100\pi$

$$SF\{\cos(n\Omega_0 t)\} = \frac{1}{2}\left[\delta(k+n) + \delta(k-n)\right]$$

$$x(t) = 2\cos(100\pi t) \overset{FS}{\longleftrightarrow} \left[\delta(k+1) + \delta(k-1)\right]$$

$$x(t-0{,}005) = 2\cos(100\pi(t-0{,}005)) \overset{FS}{\longleftrightarrow} e^{-jk\,100\pi\,.\,0{,}005}\left[\delta(k+1) + \delta(k-1)\right]$$

$$\boxed{X[k] = e^{-j\frac{k\pi}{2}}\left[\delta(k+1) + \delta(k-1)\right]}$$

c) $x(t) = -\cos(500\pi t)$, $T_0 = 1/50$

$\Omega_0 = \dfrac{2\pi}{1/50} = 100\pi$

$$SF\{\cos(n\Omega_0 t)\} = \frac{1}{2}\left[\delta(k+n) + \delta(k-n)\right]$$

$$x(t) = -\cos(500\pi t) \qquad -\cos(5\,.\,100\pi t)$$

$$\boxed{X[k] = -\frac{1}{2}\left[\delta(k+5) + \delta(k-5)\right]}$$

d) $x(t) = \frac{d}{dt}\left\{ e^{-j10\pi t} \right\}$, $T_0 = 1/5$
$\Omega_0 = \frac{2\pi}{1/5} = 10\pi$

Para $x(t) = e^{-j10\pi t}$

$$X[k] = \frac{1}{\frac{1}{5}} \int_{-\frac{1}{10}}^{1/10} e^{-j10\pi t} \cdot e^{-jk10\pi t} \, dt =$$

$$= 5 \int_{-1/10}^{1/10} e^{-j10\pi t(1+k)} \, dt = \frac{5}{-j10\pi(k+1)} \left[e^{-j10\pi(k+1)t} \right]_{-1/10}^{1/10} =$$

$$= \frac{5}{-j10\pi(k+1)} \left[e^{-j\pi(k+1)} - e^{j\pi(k+1)} \right] =$$

$$= \frac{1}{j2\pi(k+1)} \left[e^{j\pi(k+1)} - e^{-j\pi(k+1)} \right] = \frac{1}{(k+1)\pi} \left[\frac{e^{j\pi(k+1)} - e^{-j\pi(k+1)}}{2j} \right]$$

$\text{sinc}\, x = \frac{\text{sen}\, \pi x}{\pi x}$

$$= \frac{1}{(k+1)\pi} \text{sen}[\pi(k+1)] = \text{sinc}(k+1)$$

Para $x(t) = \frac{d}{dt}\left\{ e^{-j10\pi t} \right\}$

$$SF\left\{ \frac{d}{dt} x(t) \right\} = jk\Omega\, X[k]$$

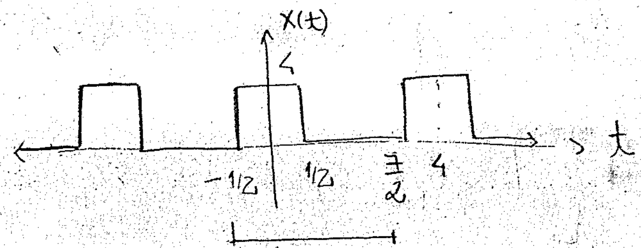
$$\boxed{X[k] = jk10\pi\, \text{sinc}(k+1)}$$

e) $x(t) = \text{rect}(t) * 4\,\delta_4(t)$, $T_0 = 4$
$\Omega_0 = \frac{2\pi}{4} = \pi/2$

$$x(t) = 4\,\text{rect}(t) * \delta_4(t)$$

$$SF\left\{ \text{rect}(t) * \delta_4(t) \right\} = \frac{\pi/2}{2\pi} \text{sinc}\left(\frac{k\pi/2}{2\pi} \right) = \frac{1}{4} \text{sinc}\left(\frac{k}{4} \right)$$

$$x(t) = 4\,\text{rect}(t) * \delta_4(t) = \text{sinc}\left(\frac{k}{4} \right)$$



$T_0 = 4$

$\Omega_0 = \frac{2\pi}{4} = \pi/2$

$$X[k] = \frac{1}{4}\int_{-1/2}^{1/2} 4\, e^{-jk\pi/2\, t}\, dt = \frac{1}{-jk\pi/2}\left[e^{-jk\pi/2\, t}\right]_{-1/2}^{1/2} =$$

$$= \frac{1}{-jk\pi/2}\left[e^{-jk\pi/4} - e^{+jk\pi/4}\right] = \frac{1}{jk\pi/2}\left[e^{jk\pi/4} - e^{-jk\pi/4}\right]\cdot\frac{4}{4} =$$

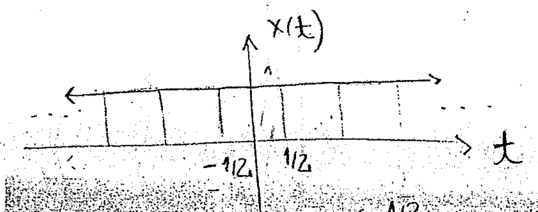
$$= \frac{4}{jk2\pi}\left[e^{jk\pi/4} - e^{-jk\pi/4}\right] = \frac{4}{k\pi}\left[\frac{e^{jk\pi/4} - e^{-jk\pi/4}}{2j}\right] =$$

$$= \frac{4}{4\cdot\frac{k\pi}{4}}\, \text{sen}\,(k\pi/4) = \frac{\text{sen}\,(k\pi/4)}{\frac{k\pi}{4}} = \boxed{\text{sinc}\left(k/4\right)}$$

f) $x(t) = \text{rect}(t) * \delta_1(t)$, $T_0 = 1$, $\Omega_0 = 2\pi$

Formulário:

$$X[k] = \frac{2\pi}{2\pi}\,\text{sinc}\left(k\,\frac{2\pi}{2\pi}\right) = \boxed{\text{sinc}(k)} = \boxed{\delta(k)}$$



$x(t) = 1$

$$X[k] = \frac{1}{1}\int_{-1/2}^{1/2} 1\, e^{-jk2\pi t}\, dt = \frac{1}{-jk2\pi}\left[e^{-jk\pi} - e^{jk\pi}\right] =$$

$$= \frac{1}{k\pi}\left[\frac{e^{jk\pi} - e^{-jk\pi}}{2j}\right] = \frac{1}{k\pi}\,\text{sen}\, k\pi = \boxed{\text{sinc}(k)}$$

g) $x(t) = \text{tri}(t) * \delta_1(t)$, $T_0 = 1$, $\Omega_0 = 2\pi$

Formulário:

$$X[k] = \frac{2\pi}{2\pi}\,\text{sinc}^2\left(k\cdot\frac{2\pi}{2\pi}\right) = \boxed{\text{sinc}^2(k)}$$

h) $x(t) = 5\left\{ tri(t-1) - tri(t+1) \right\} * \delta_4(t)$, $T_0 = 4$, $\Omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$x(t) = 5 \left\{ tri(t-1) * \delta_4(t) - tri(t+1) * \delta_4(t) \right\}$

$$SF\left\{ tri(t) * \delta_4(t) \right\} = \frac{\pi/2}{2\pi} \, sinc^2\left(k \cdot \frac{\pi/2}{2\pi} \right) = \frac{1}{4} \, sinc^2\left(\frac{k}{4} \right)$$

Prop. deslocamento no tempo:

$$x(t-t_0) \xleftrightarrow{FS} e^{-jk\Omega t_0} X[k]$$

$$X[k] = 5 \left\{ e^{-jk\pi/2} \cdot \frac{1}{4} sinc^2\left(k/4\right) - e^{jk\pi/2} \cdot \frac{1}{4} sinc^2\left(k/4\right) \right\} =$$

$$\boxed{X[k] = \frac{5}{4} sinc^2\left(k/4\right) \left(e^{-jk\pi/2} - e^{jk\pi/2} \right)}$$

i) $x(t) = \underbrace{3 sen(6\pi t)}_{n_1 = 3} + \underbrace{4 cos(8\pi t)}_{n_2 = 4}$, $T_0 = 1$, $\Omega_0 = 2\pi$

$$SF\left\{ sen(n\Omega_0 t) \right\} = \frac{j}{2} \left[\delta(k+n) - \delta(k-n) \right]$$

$$SF\left\{ cos(n\Omega_0 t) \right\} = \frac{1}{2} \left[\delta(k+n) + \delta(k-n) \right]$$

$$\boxed{X[k] = \frac{3j}{2} \left[\delta(k+3) - \delta(k-3) \right] + 2 \left[\delta(k+4) + \delta(k-4) \right]}$$

j) $x(t) = \underbrace{2 cos(24\pi t)}_{n_1 = 24} - \underbrace{8 cos(30\pi t)}_{n_2 = 30} + \underbrace{6 sen(36\pi t)}_{n_3 = 36}$, $T_0 = 2$, $\Omega_0 = \frac{2\pi}{2} = \pi$

$$\boxed{\begin{aligned} X[k] &= \left[\delta(k+24) + \delta(k-24) \right] - 4 \left[\delta(k+30) + \delta(k-30) \right] + \\ &+ 3j \left[\delta(k+36) - \delta(k-36) \right] \end{aligned}}$$

k) $x(t) = \int_{-\infty}^{t} \{ \delta_1(\lambda) - \delta_1(\lambda - 1/2) \} d\lambda$, $T_0 = 1$

$\Omega_0 = \frac{2\pi}{1} = 2\pi$

$x(t) = \int_{-\infty}^{t} \delta_1(\lambda) d\lambda - \int_{-\infty}^{t} \delta_1(\lambda - 1/2) d\lambda$

$SF\{\delta_{T_0}(t)\} = \frac{\Omega_0}{2\pi} \delta_m[k]$ $\underset{\text{período do delta}}{\overset{\text{período pedido}}{T = mT_0}}$

$SF\left\{ \int_{-\infty}^{t} x(\tau) d\tau \right\} = \frac{X[k]}{jk\Omega}$

$SF\left\{ \int_{-\infty}^{t} \delta_1(\lambda) d\lambda \right\} = \frac{\delta_1[k]}{jk2\pi} = \frac{1}{jk2\pi}$

$SF\left\{ \int_{-\infty}^{t} \delta_1(\lambda - 1/2) d\lambda \right\} = \frac{e^{-jk2\pi \cdot 1/2} \cdot 1}{jk2\pi} = \frac{e^{-jk\pi}\, 1}{jk2\pi}$

$\therefore X[k] = \frac{1}{jk2\pi} - \frac{e^{-jk\pi}\, 1}{jk2\pi} = \frac{1}{jk2\pi}\left[1 - e^{-jk\pi}\right]$

l) $x(t) = 4\cos(100\pi t)\,\text{sen}(1000\pi t)$, $T_0 = 1/50$

$\Omega_0 = \frac{2\pi}{\frac{1}{50}} = 100\pi$

$\text{sen}\,a \cos b = \frac{1}{2}[\text{sen}(a-b) + \text{sen}(a+b)]$

$\text{sen}(1000\pi t)\cos(100\pi t) = \frac{1}{2}[\text{sen}(900\pi t) + \text{sen}(1100\pi t)]$

$x(t) = \underbrace{2\,\text{sen}(900\pi t)}_{n_1 = 9} + \underbrace{2\,\text{sen}(1100\pi t)}_{n_2 = 11}$

$X[k] = j[\delta(k+9) - \delta(k-9)] + j[\delta(k+11) - \delta(k-11)]$

$$\boxed{X[k] = j[\delta(k+9) - \delta(k-9) + \delta(k+11) - \delta(k-11)]}$$

m) $x(t) = \left[14\,\mathrm{rect}(t/8) * 12\,\delta_{12}(t)\right] \otimes \left[7\,\mathrm{rect}(t/5) * 8\,\delta_8(t)\right]$, $T_0 = 24$

$\Omega_0 = \frac{2\pi}{24} = \frac{\pi}{12}$

$14 \cdot \frac{8}{8}\,\mathrm{rect}(t/8) * 12\,\delta_{12}(t) \xrightarrow{T_0 = 12} 14 \cdot 8 \cdot 12 \cdot \frac{2\pi/12}{2\pi}\,\mathrm{sinc}\left(8 \cdot k\,\frac{\cancel{2\pi}/12}{\cancel{2\pi}}\right) = 112\,\mathrm{sinc}\left(\frac{2k}{3}\right)$

$14\,\mathrm{rect}(t/8) * 12\,\delta_{12}(t) \xrightarrow{T_0 = 24} 112\,\mathrm{sinc}\left(\frac{2\,k/2}{3}\right) = 112\,\mathrm{sinc}\left(\frac{k}{3}\right)$

$7\,\mathrm{rect}(t/5) * 8\,\delta_8(t) \xrightarrow{T_0 = 8} 7 \cdot 8 \cdot 5\,\frac{2\pi/8}{2\pi}\,\mathrm{sinc}\left(5k\,\frac{\cancel{2\pi}/8}{\cancel{2\pi}}\right) = 35\,\mathrm{sinc}\left(\frac{5k}{8}\right)$

$7\,\mathrm{rect}(t/5) * 8\,\delta_8(t) \xrightarrow{T_0 = 24} 35\,\mathrm{sinc}\left(\frac{5\,k/3}{8}\right) = 35\,\mathrm{sinc}\left(\frac{5k}{24}\right)$

Pela propriedade: $x(t) \otimes y(t) \xleftrightarrow{FS} T \cdot X[k] \cdot Y[k]$

$\Rightarrow \quad X[k] = 24 \cdot \left[112\,\mathrm{sinc}\left(\frac{k}{3}\right) \cdot 35\,\mathrm{sinc}\left(\frac{5}{24}k\right)\right]$

o) $x(t) = 20\cos(40\pi t + \pi/6)$

$x(t) = 20\cos[40\pi(t + 1/240)]$

Para $x_1(t) = 20\cos(40\pi t)$, $\Omega_0 = 40\pi = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{40\pi} = \frac{1}{20}$

$X_1[k] = 10[\delta(k+1) + \delta(k-1)]$

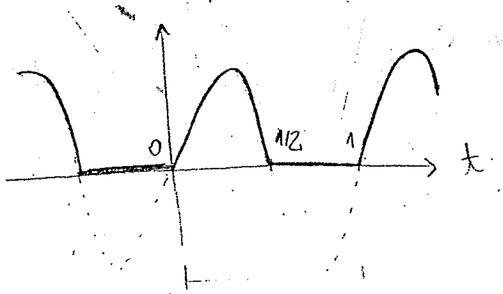
Propriedade:

$x(t-t_0) \xleftrightarrow{FS} e^{-jk\Omega t_0} X[k]$

$X[k] = e^{-jk40\pi \cdot -1/240} \cdot 10 [\delta(k+1) + \delta(k-1)] =$

$$\boxed{X[k] = 10\, e^{jk\pi/6} [\delta(k+1) + \delta(k-1)]}$$

p) $x(t) = \text{sen}(2\pi t)$



$T_0 = 1$

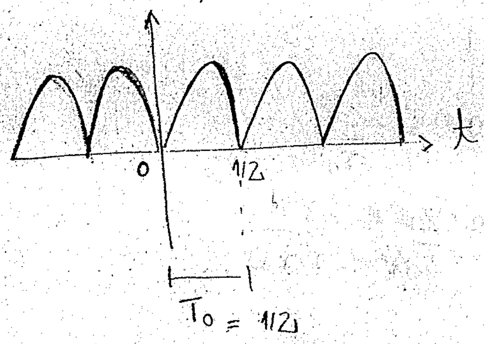
$\Omega_0 = 2\pi$

$X[k] = \frac{1}{1} \int_0^{1/2} \text{sen}\, 2\pi t\, e^{-jk2\pi t} dt = \int_0^{1/2} \left(\frac{e^{j2\pi t} - e^{-j2\pi t}}{2j} \right) e^{-jk2\pi t} dt =$

$= \frac{1}{2j} \left[\int_0^{1/2} e^{-j2\pi t(k-1)} dt - \int_0^{1/2} e^{-j2\pi t(k+1)} dt \right] =$

$= \frac{1}{2j} \left[\frac{1}{-j2\pi(k-1)} (e^{-j\pi(k-1)} - 1) + \frac{1}{j2\pi(k+1)} (e^{-j\pi(k+1)} - 1) \right]$

9) $x(t) = |sen(2\pi t)|$



$T_0 = 1/2$

$\Omega_0 = \frac{2\pi}{1/2} = 4\pi$

$$X[k] = \frac{1}{1/2}\int_0^{1/2} sen\, 2\pi t\, e^{-jk4\pi t}\,dt = 2\int_0^{1/2}\left(\frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}\right)e^{-jk4\pi t}\,dt$$

$$= \frac{1}{j}\left[\int_0^{1/2} e^{j2\pi t(1-2k)}\,dt - \int_0^{1/2} e^{-j2\pi t(1+2k)}\,dt\right] =$$

$$= \frac{1}{j}\left[\frac{1}{j2\pi(1-2k)}\left(e^{j\pi(1-2k)} - 1\right) + \frac{1}{j2\pi(1+2k)}\left(e^{-j\pi(1+2k)} - 1\right)\right]$$

$$\frac{1}{j}\left[\frac{\overbrace{e^{j\pi}}^{-1}\cdot\overbrace{e^{-j2k\pi}}^{1} - 1}{j2\pi - j4\pi k} + \frac{\overbrace{e^{-j\pi}}^{-1}\cdot\overbrace{e^{-j2k\pi}}^{1} - 1}{j2\pi + j4\pi}\right]$$

$$\frac{-2}{j}\left[\frac{1}{j2\pi(1-2k)} + \frac{1}{j2\pi(1+2k)}\right]$$

$$\frac{-2}{-2\pi}\left\{\frac{1}{1-2k} + \frac{1}{1+2k}\right\}$$

$$\boxed{X[k] = \frac{1}{\pi}\left\{\frac{1}{1-2k} + \frac{1}{1+2k}\right\}}$$

continuação p)

$$= \frac{1}{2j}\left[\frac{1}{-j2\pi(k-1)}\left(e^{-j\pi(k-1)}-1\right)+\frac{1}{j2\pi(k+1)}\left(e^{-j\pi(k+1)}-1\right)\right]$$

$$= \frac{1}{2j}\left[\frac{e^{j\pi}\cdot e^{-jk\pi}-1}{-j2\pi(k-1)}+\frac{e^{-jk\pi}\cdot e^{-j\pi}-1}{j2\pi(k+1)}\right]$$

$$= \frac{1}{2j}\left[\frac{-e^{-jk\pi}-1}{-j2\pi(k-1)}-\frac{-e^{-jk\pi}-1}{j2\pi(k+1)}\right]$$

$$= \frac{1}{2}\left[\frac{e^{-jk\pi}+1}{-2\pi(k-1)}+\frac{e^{-jk\pi}+1}{2\pi(k+1)}\right]$$

↓ para correro multiplica por $e^{jk\pi/2}$

$$\frac{1}{2}\left[\frac{e^{-jk\pi+jk\pi/2}+e^{jk\pi/2}}{-2\pi(k-1)}+\frac{e^{-jk\pi+jk\pi/2}+e^{jk\pi/2}}{2\pi(k+1)}\right]=$$

$$=\frac{1}{2}\left[\frac{e^{-jk\pi/2}+e^{jk\pi/2}}{-2\pi(k-1)}+\frac{e^{-jk\pi/2}+e^{jk\pi/2}}{2\pi(k+1)}\right]$$

$e^{-jk\Omega t_0}$ $\quad \Omega=2\pi$

$e^{-jk\Omega/4}$ $= e^{-jk\pi/2}$

$e^{-jk2\pi/4}$

4)

a) $x[k] = \delta[k-2] + \delta[k] + \delta[k+2]$, $T_0 = 1$
$\Omega_0 = 2\pi$

$$SF\{\cos(n\Omega_0 t)\} = \frac{1}{2}[\delta(k+n) + \delta(k-n)]$$

$\underline{n = 2}$

$\delta[k-2] + \delta[k+2] \longrightarrow 2\cos(4\pi t)$

$\delta[k] \longrightarrow ?$ $\quad \cos(\phi t) = 1$

$$\boxed{x(t) = 2\cos(4\pi t) + 1}$$

b) $x[k] = 10\,\mathrm{sinc}\left(\frac{k}{10}\right)$, $T_0 = 1$
$\Omega_0 = 2\pi$

$$SF\left\{10 \cdot \frac{1}{1/10}\,\mathrm{rect}\left(\frac{t}{1/10}\right) * \delta_1(t)\right\} = 10 \cdot SF\left\{\frac{1}{1/10}\,\mathrm{rect}\left(\frac{t}{1/10}\right) * \delta_1(t)\right\} =$$

$$= 10 \cdot \frac{\cancel{2\pi}}{\cancel{2\pi}}\,\mathrm{sinc}\left(\frac{1}{10} \cdot k \cdot \frac{\cancel{2\pi}}{\cancel{2\pi}}\right) = 10\,\mathrm{sinc}\left(\frac{k}{10}\right)$$

$$\therefore \boxed{x(t) = 100 \cdot \mathrm{rect}(10t) * \delta_1(t)}$$

$T = T_F$

$$\frac{\delta[k+1] + \delta[k-1]}{2} \qquad \cos\left(\frac{2\pi k_1}{T_F}t\right)$$

…cición de Transformada de Fourier

a) $x(t) = 10 \operatorname{sen} t$

$TF\{\operatorname{sen}(\Omega_0 t)\} = j\pi[\delta(\Omega+\Omega_0) - \delta(\Omega-\Omega_0)]$

$TF\{10 \operatorname{sen} t\} = \boxed{j10\pi[\delta(\Omega+1) - \delta(\Omega-1)]}$

b) $x(t) = 10 \operatorname{sen}(t-2)$

$TF\{10 \operatorname{sen}(t-2)\} = \boxed{j10\pi[\delta(\Omega+1) - \delta(\Omega-1)]\, e^{-j2\Omega}}$

c) $x(t) = 10 \operatorname{sen}(2(t-1))$

$TF\{10 \operatorname{sen}(2(t-1))\} = \boxed{j10\pi[\delta(\Omega+2) - \delta(\Omega-2)] \cdot e^{-j\Omega}}$

D) $x(t) = 10 \operatorname{sen}(2t-1) =$

$= 10 \operatorname{sen}(2(t-1/2))$

$TF\{10 \operatorname{sen}(2(t-1/2))\} = \boxed{j10\pi[\delta(\Omega+2) - \delta(\Omega-2)]\, e^{-j\frac{\Omega}{2}}}$

e) $x(t) = 5 \operatorname{rect}(2t-1) = 5 \operatorname{rect}(2(t-1/2))$

$TF\{5 \operatorname{rect}(2(t-1/2))\} = \boxed{\frac{5}{2} \operatorname{sinc}\left(\frac{\Omega}{4\pi}\right) e^{-j\Omega/2}}$

f) $x(t) = 5 \operatorname{rect}((t/2)-1) = 5 \operatorname{rect}(1/2(t-2))$

$TF\{5 \operatorname{rect}(1/2(t-2))\} = \frac{5}{1/2} \operatorname{sinc}\left(\frac{\Omega/1/2}{2\pi}\right) e^{-j2\Omega} =$

$= \boxed{10 \operatorname{sinc}\left(\frac{\Omega}{\pi}\right) e^{-j2\Omega}}$

g) $x(t) = 5 \operatorname{rect}(2(t-1))$

$5 \operatorname{rect}(t) \overset{FT}{\longleftrightarrow} 5 \operatorname{sinc}\left(\frac{\Omega}{2\pi}\right)$

$5 \operatorname{rect}(2t) \overset{FT}{\longleftrightarrow} \frac{5}{2} \operatorname{sinc}\left(\frac{\Omega}{4\pi}\right)$

$5 \operatorname{rect}(2(t-1)) \overset{FT}{\longleftrightarrow} \frac{5}{2} \operatorname{sinc}\left(\frac{\Omega}{4\pi}\right) e^{-j\Omega}$

h) $x(t) = 5\,\mathrm{rect}\,((t-1)/2)$

$$5\,\mathrm{rect}\,(t) \overset{FT}{\longleftrightarrow} 5\,\mathrm{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$5\,\mathrm{rect}\,(t/2) \overset{FT}{\longleftrightarrow} 10\,\mathrm{sinc}\left(\frac{\Omega}{\pi}\right)$$

$$5\,\mathrm{rect}\left(\frac{(t-1)}{2}\right) \overset{FT}{\longleftrightarrow} \boxed{10\,\mathrm{sinc}\left(\frac{\Omega}{\pi}\right) e^{-j\Omega}}$$

i) $x(t) = \mathrm{tri}(t)$

$$\mathrm{tri}(t) \overset{FT}{\longleftrightarrow} \boxed{\mathrm{sinc}^2\left(\frac{\Omega}{2\pi}\right)}$$

j) $x(t) = 5\,\mathrm{sen}\,(3t-(\pi/4)) = 5\,\mathrm{sen}\,(3\,(t-\pi/12))$

$$5\,\mathrm{sen}\,(3t) \overset{FT}{\longleftrightarrow} j5\pi\,[\delta(\Omega+3)-\delta(\Omega-3)]$$

$$5\,\mathrm{sen}\,(3(t-\pi/12)) \overset{FT}{\longleftrightarrow} \boxed{j5\pi\,[\delta(\Omega+3)-\delta(\Omega-3)]\,e^{-j\Omega\pi/12}}$$

k) $x(t) = 5\,\mathrm{sen}\,(3(t+1))$

$$5\,\mathrm{sen}\,(3(t+1)) \overset{FT}{\longleftrightarrow} \boxed{j5\pi\,[\delta(\Omega+3)-\delta(\Omega-3)]\,e^{+j\Omega}}$$

l) $x(t) = 5\,\mathrm{sen}\,((t/3)-(\pi/4)) = 5\,\mathrm{sen}\left(\frac{1}{3}(t-3\pi/4)\right)$

$$\frac{t}{3}-\frac{\pi}{4} = \frac{4t-3\pi}{12} = \frac{1}{12}(4t-3\pi) = \frac{4}{12}(t-3\pi/4) = \frac{1}{3}(t-3\pi/4)$$

$$5\,\mathrm{sen}\left(\frac{1}{3}t\right) \overset{FT}{\longleftrightarrow} j5\pi\,[\delta(\Omega+1/3)-(\Omega-1/3)]$$

$$5\,\mathrm{sen}\left(\frac{1}{3}\left(t-\frac{3\pi}{4}\right)\right) \overset{FT}{\longleftrightarrow} \boxed{j5\pi\,[\delta(\Omega+1/3)-(\Omega-1/3)]\,e^{-j\Omega 3\pi/4}}$$

l) $x(t) = 5\,\text{sen}\,((t+1)/3)$

$$5\,\text{sen}\left(\frac{1}{3}t\right) \xleftrightarrow{FT} j5\pi\left[\delta(\Omega+1/3) - \delta(\Omega-1/3)\right]$$

$$5\,\text{sen}\left(\frac{(t+1)}{3}\right) \xleftrightarrow{FT} \boxed{j5\pi\left[\delta(\Omega+1/3) - \delta(\Omega-1/3)\right] e^{j\Omega}}$$

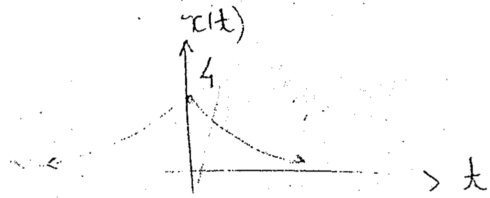
m) $x(t) = \delta(t+1/2) + \delta(t-1/2)$

$$\delta(t+1/2) + \delta(t-1/2) \xleftrightarrow{FT} 2\, e^{+j\Omega/2} + e^{-j\Omega/2} = \boxed{2\cos(\Omega/2)}$$

o) $x(t) = \delta(t-1) - \delta(t+1)$

$$\delta(t-1) - \delta(t+1) \xleftrightarrow{FT} \boxed{e^{-j\Omega} - e^{j\Omega}}$$

p) $x(t) = 4\,e^{-|t|/16}$



$$Y(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{-\infty}^{0} 4\, e^{t/16}\, e^{-j\Omega t}\, dt + \int_{0}^{\infty} 4\, e^{-t/16}\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{0} 4\, e^{t(1/16 - j\Omega)}\, dt + \int_{0}^{\infty} 4\, e^{-t(1/16 + j\Omega)}\, dt =$$

$$= \frac{4}{\frac{1}{16} - j\Omega}\, e^{t(1/16 - j\Omega)}\Big|_{-\infty}^{0} - \frac{4}{\frac{1}{16} + j\Omega}\, e^{-t(1/16 + j\Omega)}\Big|_{0}^{\infty}$$

$$= \frac{4}{\frac{1}{16} - j\Omega} + \frac{4}{\frac{1}{16} + j\Omega} = \frac{4(1/16 + j\Omega) + 4(1/16 - j\Omega)}{\left(\frac{1}{16}\right)^2 + \Omega^2} =$$

$$(a+bi)(a-bi) = a^2 - abi + abi - b^2 i^2 = a^2 + b^2$$

$$= \frac{4 \cdot 1/16 + \cancel{4j\Omega} + 4 \cdot \frac{1}{16} - \cancel{4j\Omega}}{(1/16)^2 + \Omega^2} =$$

$$= \frac{2 \cdot 4/16}{(1/16)^2 + \Omega^2} = \boxed{\frac{1/2}{(1/16)^2 + \Omega^2}}$$

q) $x(t) = [\, 2e^{(-1+j2\pi)t}\, u(t) + 2e^{(-1-j2\pi)t}\, u(t) \,]$

$TF\{ e^{-at} u(t) \} = \dfrac{1}{a+j\Omega}$

$[\, 2e^{-(1-j2\pi)t} + 2e^{-(+1+j2\pi)t} \,] \overset{FT}{\longleftrightarrow} \boxed{2\left[\dfrac{1}{1+j(\Omega-2\pi)} + \dfrac{1}{1+j(\Omega+2}\right.}$

r) $x(t) = 2\,\delta_2(t) - 2\,\delta_2(t-1)$

$TF\{ \delta_{T_0}(t) \} = \Omega_0\, \delta_{\Omega_0}(\Omega) \qquad \Omega_0 = 2\pi/T_0$

$2\,\delta_2(t) \overset{FT}{\longleftrightarrow} 2\pi\, \delta_\pi(\Omega)$

$-2\,\delta_2(t-1) \overset{FT}{\longleftrightarrow} -2\pi\, \delta_\pi(\Omega)\, e^{-j\Omega}$

$TF\{x(t)\} = \boxed{2\pi\, \delta_\pi(\Omega)\, (1 - e^{-j\Omega})}$

s) $x(t) = 10\,\mathrm{sen}(t) * 2\,\delta(t+4)$

$10\,\mathrm{sen}(t) \overset{FT}{\longleftrightarrow} 10\pi\,[\,\delta(\Omega+1) - \delta(\Omega-1)\,]$

$2\,\delta(t+4) \overset{FT}{\longleftrightarrow} 2\,e^{j4\Omega}$

$TF\{x(t)\} = \boxed{j\,20\pi\,[\,\delta(\Omega+1) - \delta(\Omega-1)\,]\, e^{j4\Omega}}$

$TF^{-1}\{X(j\Omega)\} = 20\,\mathrm{sen}(t+4)$ ⇒ resultado da convolução ?

t) $x(t) = \mathrm{rect}(t) * \delta_2(t)$

$\mathrm{rect}(t) \overset{FT}{\longleftrightarrow} \mathrm{sinc}\left(\dfrac{\Omega}{2\pi}\right)$

$\delta_2(t) \overset{FT}{\longleftrightarrow} \pi\, \delta_\pi(\Omega)$

$TF\{x(t)\} = \pi\, \mathrm{sinc}\left(\dfrac{\Omega}{2\pi}\right) \delta_\pi(\Omega)$

u) $x(t) = tri(10t) * \delta_{1/4}(t)$

$$tri(10t) \overset{FT}{\longleftrightarrow} \frac{1}{10}\, sinc^2\left(\frac{\Omega}{20\pi}\right)$$

$$\delta_{1/4}(t) \overset{FT}{\longleftrightarrow} 8\pi\, \delta_{8\pi}(\Omega)$$

$$\Omega_0 = \frac{2\pi}{1/4} = 8\pi$$

$$TF\{x(t)\} = \boxed{\frac{8\pi}{10}\, sinc^2\left(\frac{\Omega}{20\pi}\right)\delta_{8\pi}(\Omega)}$$

v) $x(t) = 5\, sinc(2t-1) = 5\, sinc(2(t-1/2))$

$$rect(t) \overset{FT}{\longleftrightarrow} sinc\left(\frac{\Omega}{2\pi}\right)$$

<u>Dualidade</u>

$x(t) \overset{FT}{\longleftrightarrow} X(j\Omega)$

$X(jt) \overset{FT}{\longleftrightarrow} 2\pi\, x(-\Omega)$

$$sinc\left(\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\pi\, rect(-\Omega) = 2\pi\, rect(\Omega)$$

$$sinc\left(4\pi\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} \frac{2\pi}{4\pi}\, rect\left(\frac{\Omega}{4\pi}\right)$$

$$sinc(2t) \overset{FT}{\longleftrightarrow} \frac{1}{2}\, rect\left(\frac{\Omega}{4\pi}\right)$$

$$5\, sinc(2(t-1/2)) \overset{FT}{\longleftrightarrow} \boxed{\frac{5}{2}\, rect\left(\frac{\Omega}{4\pi}\right) e^{-j\Omega/2}}$$

w) $x(t) = 5\, sinc((t/2)-1) = 5\, sinc\left(\tfrac{1}{2}(t-2)\right)$

$$\frac{t}{2} - \frac{2}{2} = \frac{1}{2}(t-2)$$

$$rect(t) \overset{FT}{\longleftrightarrow} sinc\left(\frac{\Omega}{2\pi}\right)$$

$$sinc\left(\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\pi\, rect(-\Omega) = 2\pi\, rect(\Omega)$$

$$sinc\left(\pi\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\, rect\left(\frac{\Omega}{\pi}\right)$$

$$5\, sinc\left(\frac{1}{2}(t-2)\right) \overset{FT}{\longleftrightarrow} 10\, rect\left(\frac{\Omega}{\pi}\right) e^{-j2\Omega}$$

x) $x(t) = 5\,\mathrm{sinc}(2(t-1))$

$$\mathrm{sinc}(2t) \overset{FT}{\longleftrightarrow} \frac{1}{2}\,\mathrm{rect}\left(\frac{\Omega}{4\pi}\right)$$

$$5\,\mathrm{sinc}(2(t-1)) \overset{FT}{\longleftrightarrow} \boxed{\frac{5}{2}\,\mathrm{rect}\left(\frac{\Omega}{4\pi}\right) e^{-j\Omega}}$$

y) $x(t) = 5\,\mathrm{sinc}((t-1)/2)$

$$\mathrm{sinc}\left(\tfrac{1}{2}t\right) \overset{FT}{\longleftrightarrow} 2\,\mathrm{rect}\left(\frac{\Omega}{\pi}\right)$$

$$5\,\mathrm{sinc}\left(\frac{1}{2}(t-1)\right) \overset{FT}{\longleftrightarrow} \boxed{10\,\mathrm{rect}\left(\frac{\Omega}{\pi}\right) e^{-j\Omega}}$$

z) $$\mathrm{rect}(t) \overset{FT}{\longleftrightarrow} \mathrm{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\mathrm{sinc}\left(\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\pi\,\mathrm{rect}(\Omega)$$

$$\mathrm{sinc}\left(8\pi\frac{t}{2\pi}\right) = \mathrm{sinc}(4t) \overset{FT}{\longleftrightarrow} \frac{1}{4}\,\mathrm{rect}\left(\frac{\Omega}{8\pi}\right)$$

$$4\,\mathrm{sinc}(4t) \overset{FT}{\longleftrightarrow} \mathrm{rect}\left(\frac{\Omega}{8\pi}\right)$$

$$-2\,\mathrm{sinc}(4(t-1/4)) \overset{FT}{\longleftrightarrow} -\frac{1}{2}\,\mathrm{rect}\left(\frac{\Omega}{8\pi}\right) e^{-j\Omega/4}$$

$$-2\,\mathrm{sinc}(4(t+1/4)) \overset{FT}{\longleftrightarrow} -\frac{1}{2}\,\mathrm{rect}\left(\frac{\Omega}{8\pi}\right) e^{j\Omega/4}$$

$$X(j\Omega) = \mathrm{rect}\left(\frac{\Omega}{8\pi}\right) - \frac{1}{2}\,\mathrm{rect}\left(\frac{\Omega}{8\pi}\right)\left[2\frac{(e^{j\Omega/4} + e^{-j\Omega/4})}{2}\right] =$$

$$= \mathrm{rect}\left(\frac{\Omega}{8\pi}\right) - \mathrm{rect}\left(\frac{\Omega}{8\pi}\right)\cos(\Omega/4) =$$

$$= \boxed{\mathrm{rect}\left(\frac{\Omega}{8\pi}\right)\left[1 - \cos\left(\frac{\Omega}{4}\right)\right]}$$

2)

a) $x(t) = \text{rect}(t) * \cos(\pi t)$

$$\text{rect}(t) \overset{TF}{\longleftrightarrow} \text{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\cos(\pi t) \overset{TF}{\longleftrightarrow} \pi\left[\delta(\Omega+\pi) + \delta(\Omega-\pi)\right]$$

$$X(j\Omega) = \pi\, \text{sinc}\left(\frac{\Omega}{2\pi}\right)\left[\delta(\Omega+\pi) + \delta(\Omega-\pi)\right]$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\Omega)\, e^{+j\Omega t}\, d\Omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \pi\, \text{sinc}\left(\frac{\Omega}{2\pi}\right)\left[\delta(\Omega+\pi)+\delta(\Omega-\pi)\right] e^{+j\Omega t}\, d\Omega$$

$$\frac{1}{2}\left[\int_{-\infty}^{\infty} \text{sinc}\left(\frac{\Omega}{2\pi}\right) e^{j\Omega t}\, \delta(\Omega+\pi)\, d\Omega + \int_{-\infty}^{\infty} \text{sinc}\left(\frac{\Omega}{2\pi}\right) e^{j\Omega t}\, \delta(\Omega-\pi)\, d\Omega\right] =$$

$$\boxed{\begin{gathered} x(t) * \delta(t-a) = x(t-a) \\ \Downarrow \\ \int_{\substack{\text{intervalo} \\ t=a}} f(t)\,\delta(t-a) = f(a) \end{gathered}}$$

$$\text{sinc}\, x = \frac{\text{sen}\, \pi x}{\pi x}$$

$$\text{sinc}(-1/2) = \frac{\text{sen} -\pi/2}{-\pi/2} = \frac{-1}{-\pi/2} = \frac{2}{\pi}$$

$$\text{sinc}(1/2) = 1/\pi/2 = 2/\pi$$

$$= \frac{1}{2}\left[\text{sinc}\left(\frac{-\pi}{2\pi}\right) e^{-j\pi t} + \text{sinc}\left(\frac{\pi}{2\pi}\right) e^{j\pi t}\right] = \frac{1}{2}\left[\frac{2}{\pi} e^{-j\pi t} + \frac{2}{\pi} e^{j\pi t}\right] =$$

$$= \frac{2}{\pi}\left[\frac{e^{-j\pi t} + e^{j\pi t}}{2}\right] = \frac{2}{\pi}\left[\frac{e^{j\pi t} + e^{-j\pi t}}{2}\right] =$$

$$\boxed{x(t) = \frac{2}{\pi}\cos(\pi t)}$$

b) $x(t) = \text{rect}(t) * \cos(2\pi t)$

$$\text{rect}(t) \xleftrightarrow{FT} \text{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\cos(2\pi t) \xleftrightarrow{FT} \pi\left[\delta(\Omega + 2\pi) + \delta(\Omega - 2\pi)\right]$$

$$X(j\Omega) = \pi\, \text{sinc}\left(\frac{\Omega}{2\pi}\right)\left[\delta(\Omega+2\pi) + \delta(\Omega - 2\pi)\right]$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega =$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} \text{sinc}\left(\frac{\Omega}{2\pi}\right) e^{j\Omega t}\,\delta(\Omega+2\pi)\, d\Omega + \int_{-\infty}^{\infty} \text{sinc}\left(\frac{\Omega}{2\pi}\right) e^{j\Omega t}\,\delta(\Omega-2\pi)\, d\Omega\right]$$

$$= \frac{1}{2}\left[\underbrace{\text{sinc}\left(-\frac{2\pi}{2\pi}\right)}_{\phi} e^{-j2\pi t} + \underbrace{\text{sinc}\left(\frac{2\pi}{2\pi}\right)}_{\phi} e^{j2\pi t}\right] = 0$$

$$\text{sinc}(-1) = \frac{\text{sen} -\pi}{-\pi} = \phi$$

$$\boxed{x(t) = 0}$$

$$\text{sinc}(1) = \frac{\text{sen}\, \pi}{\pi} = \phi$$

c) $x(t) = \text{sinc}(t) * \text{sinc}(t/2)$

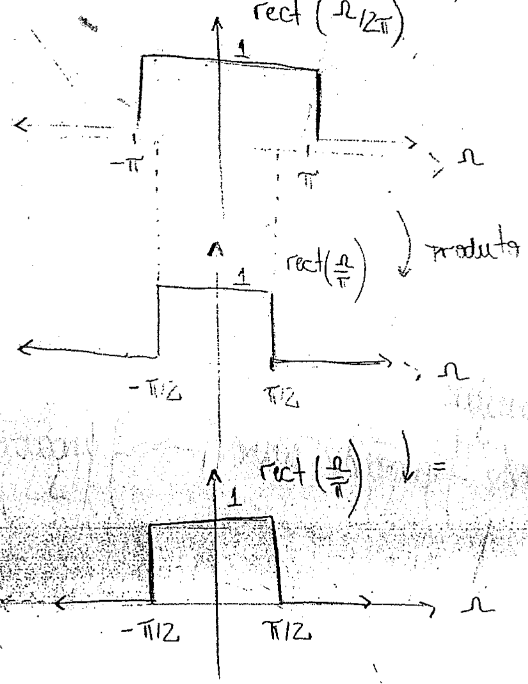
$$\text{rect}(t) \xleftrightarrow{FT} \text{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\text{sinc}\left(\frac{t}{2\pi}\right) \xleftrightarrow{FT} 2\pi\, \text{rect}(\Omega)$$

$$\text{sinc}\left(2\pi \cdot \frac{t}{2\pi}\right) \xleftrightarrow{FT} \text{rect}\left(\frac{\Omega}{2\pi}\right)$$

$$\begin{cases} \text{sinc}(t) \xleftrightarrow{FT} \text{rect}\left(\frac{\Omega}{2\pi}\right) \\ \text{sinc}\left(\frac{t}{2}\right) \xleftrightarrow{FT} 2\, \text{rect}\left(\frac{\Omega}{\pi}\right) \end{cases}$$

$$X(j\Omega) = 2\, \text{rect}\left(\frac{\Omega}{2\pi}\right) \text{rect}\left(\frac{\Omega}{\pi}\right) = 2\, \text{rect}\left(\frac{\Omega}{\pi}\right)$$



$$\boxed{X^{-1}(j\Omega) = x(t) = \text{sinc}\left(t/2\right)}$$

D) $x(t) = \text{sinc}(t) * \text{sinc}^2(t/2)$

$\Rightarrow \text{sinc}(t) \xleftrightarrow{FT} \text{rect}\left(\frac{\Omega}{2\pi}\right)$

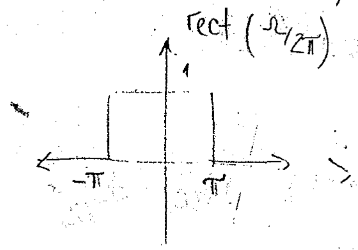
$\text{tri}(t) \xleftrightarrow{FT} \text{sinc}^2\left(\frac{\Omega}{2\pi}\right)$

$\text{sinc}^2\left(\frac{t}{2\pi}\right) \xleftrightarrow{FT} 2\pi \, \text{tri}(\Omega)$

$\text{sinc}^2\left(\pi \frac{t}{2\pi}\right) \xleftrightarrow{FT} 2 \, \text{tri}\left(\frac{\Omega}{\pi}\right)$

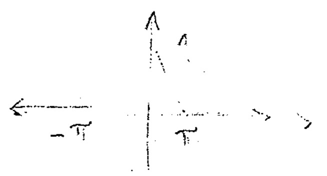
$\text{sinc}^2\left(\frac{t}{2}\right) \xleftrightarrow{FT} 2 \, \text{tri}\left(\frac{\Omega}{\pi}\right)$

$X(j\Omega) = 2 \, \text{rect}\left(\frac{\Omega}{2\pi}\right) \text{tri}\left(\frac{\Omega}{\pi}\right) = 2 \, \text{tri}\left(\frac{\Omega}{\pi}\right) \rightarrow \boxed{x(t) = \text{sinc}^2(t/2)}$



$\frac{\Omega}{2\pi} = \frac{1}{2}$

$\Omega = \pi$



$\frac{\Omega}{\pi} = 1$

e) $x(t) = e^{-t} u(t) * \text{sen}(2\pi t)$

convolução duas funções reais $\rightarrow$ função real

$e^{-t} u(t) \xleftrightarrow{FT} \frac{1}{1 + j\Omega}$

$\text{sen}(2\pi t) \xleftrightarrow{FT} j\pi \left[\delta(\Omega + 2\pi) - \delta(\Omega - 2\pi) \right]$

$X(j\Omega) = j\pi \frac{1}{1 + j\Omega} \left[\delta(\Omega + 2\pi) - \delta(\Omega - 2\pi) \right]$

$X^{-1}(j\Omega) = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} \, d\Omega =$

$= \frac{j}{2} \left[\int_{-\infty}^{\infty} \frac{1}{1 + j\Omega} e^{j\Omega t} \delta(\Omega + 2\pi) \, d\Omega - \int_{-\infty}^{\infty} \frac{1}{1 + j\Omega} e^{j\Omega t} \delta(\Omega - 2\pi) \, d\Omega \right] =$

$= \boxed{\frac{j}{2} \left[\frac{1}{1 - j2\pi} e^{-j2\pi t} - \frac{1}{1 + j2\pi} e^{j2\pi t} \right]}$

$$= \frac{j}{2}\left[\frac{(1+j2\pi)e^{-j2\pi t} - (1-j2\pi)e^{j2\pi t}}{(1-j2\pi)(1+j2\pi)}\right]$$

$$= \frac{j}{2}\left[\frac{(1+j2\pi)(\cos 2\pi t - j\,\text{sen}\,2\pi t) - (1-j2\pi)(\cos 2\pi t + j\,\text{sen}\,2\pi t)}{(1+4\pi^2)}\right]$$

$$= \frac{j}{2(1+4\pi^2)}\left[\cancel{\cos 2\pi t} - j\,\text{sen}\,2\pi t + j2\pi\cos 2\pi t + \cancel{2\pi\,\text{sen}\,2\pi t} - \cancel{\cos 2\pi t} - j\,\text{sen}\,2\pi t + j2\pi\cos 2\pi t - \cancel{2\pi\,\text{sen}\,2\pi t}\right]$$

$$= \frac{j}{2(1+4\pi^2)}\left[-\cancel{2}j\,\text{sen}\,2\pi t + \cancel{2}\,j2\pi\cos 2\pi t\right] =$$

$$= \boxed{\frac{\text{sen}\,2\pi t - 2\pi\cos 2\pi t}{1+4\pi^2}}$$

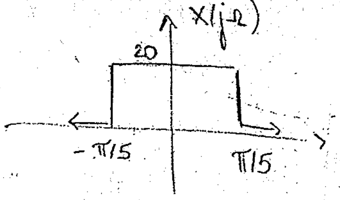
3)

a) $x(t) = 4\,\text{sinc}(t/5)$

$$\text{sinc}(t) \overset{FT}{\longleftrightarrow} \text{rect}\left(\frac{\Omega}{2\pi}\right)$$

$$\text{sinc}\left(\frac{t}{5}\right) \overset{FT}{\longleftrightarrow} 5\,\text{rect}\left(\frac{5\Omega}{2\pi}\right)$$

$$4\,\text{sinc}(t/5) \overset{FT}{\longleftrightarrow} 20\,\text{rect}\left(\frac{5\Omega}{2\pi}\right)$$



$$\frac{5\Omega}{2\pi} = \frac{1}{2} \qquad \Omega = \frac{\pi}{5}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega =$$

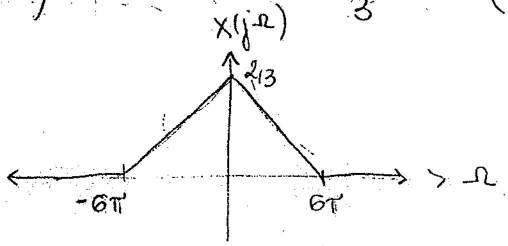
$$= \frac{1}{2\pi}\int_{-\pi/5}^{\pi/5} 20^2\, d\Omega = \frac{400}{\cancel{2\pi}} \cdot \frac{\cancel{2\pi}}{5} = 80$$

b) $x(t) = 2\,\text{sinc}^2(3t)$

$$\text{sinc}^2\left(\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\pi\,\text{tri}(\Omega)$$

$$\text{sinc}^2\left(6\pi \cdot \frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} \frac{1}{3}\,\text{tri}\left(\frac{\Omega}{6\pi}\right)$$

$$2\,\text{sinc}^2(3t) \overset{FT}{\longleftrightarrow} \frac{2}{3}\,\text{tri}\left(\Omega/6\pi\right)$$



$$\frac{\Omega}{6\pi} = 1$$

$$Y - Y_0 = m(x - x_0)$$
$$0 - 2/3 = m(6\pi - 0)$$
$$-2/3 = m\,6\pi$$
$$m = -\frac{\frac{2}{3}}{6\pi} = -\frac{\cancel{2}}{18\pi} = -\frac{1}{9\pi}$$
$$Y - 2/3 = -\frac{1}{9\pi}x$$
$$Y = -\frac{1}{9\pi}x + 2/3$$
$$Y = -\frac{1}{9\pi}(x - 6\pi)$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega =$$

$$= \frac{1}{2\pi} \cdot 2 \int_0^{6\pi} \left(-1/9\pi\,(\Omega - 6\pi)\right)^2 d\Omega =$$

$$= \frac{1}{\pi} \cdot \frac{1}{(9\pi)^2} \cdot \frac{(\Omega - 6\pi)^3}{3}\Bigg|_0^{6\pi} = \frac{1}{243\pi^3} \cdot -(-6\pi)^3 =$$

4)

a) $X(j\Omega) = e^{-4\Omega^2}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-4\Omega^2} \cdot e^{j\Omega t}\, d\Omega$$

b) $X(j\Omega) = 7\,\mathrm{sinc}^2\left(\frac{\Omega}{\pi}\right) \longrightarrow \boxed{X^{-1}(j\Omega) = \frac{7}{2}\,\mathrm{tri}(t/2)}$

$$\mathrm{tri}(t) \overset{FT}{\longleftrightarrow} \mathrm{sinc}^2\left(\frac{\Omega}{2\pi}\right)$$

$$\mathrm{tri}(t/2) \overset{FT}{\longleftrightarrow} 2\,\mathrm{sinc}^2\left(\frac{\Omega}{\pi}\right)$$

$$\frac{7}{2}\,\mathrm{tri}(t/2) \overset{FT}{\longleftrightarrow} 7\,\mathrm{sinc}^2\left(\frac{\Omega}{\pi}\right)$$

c) $X(j\Omega) = j\pi\left[\delta(\Omega+10\pi) - \delta(\Omega-10\pi)\right] \Rightarrow \boxed{X^{-1}(j\Omega) = \mathrm{sen}\,(10\pi t)}$

$$\mathrm{sen}\,(\Omega_0 t) \overset{FT}{\longleftrightarrow} j\pi\left[\delta(\Omega+\Omega_0) - \delta(\Omega-\Omega_0)\right]$$

D) $X(j\Omega) = \frac{\pi}{20}\,\delta_{1/4}(\Omega)$ $\Omega_0 = 1/4$ $\Omega_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{1/4} = 8\pi$

$$\delta_{8\pi}(t) \overset{FT}{\longleftrightarrow} \frac{1}{4}\,\delta_{1/4}(\Omega)$$

$\Omega_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$

$$\frac{\pi}{5}\,\delta_{8\pi}(t) \overset{FT}{\longleftrightarrow} \frac{\pi}{5}\cdot\frac{1}{4}\,\delta_{1/4}(\Omega)$$

$$\frac{\pi}{5}\,\delta_{8\pi}(t) \quad \frac{\pi}{20}\,\delta_{1/4}(\Omega) \quad \rightarrow \quad \boxed{X^{-1}(j\Omega) = \frac{\pi}{5}\,\delta_{8\pi}(t)}$$

e) $X(j\Omega) = \frac{5\pi}{j\Omega} + 10\pi\,\delta(\Omega) \rightarrow \boxed{X^{-1}(j\Omega) = \frac{5\pi}{2}\,\mathrm{sgn}(t) + 5}$

$\mathrm{sgn}(t) \overset{FT}{\longleftrightarrow} \frac{2}{j\Omega}$ | $1 \overset{FT}{\longleftrightarrow} 2\pi\,\delta(\Omega)$

$5 \overset{FT}{\longleftrightarrow} 10\pi\,\delta(\Omega)$

$\frac{5\pi}{2}\,\mathrm{sgn}(t) \overset{FT}{\longleftrightarrow} \frac{5\pi}{j\Omega}$

f) $X(j\Omega) = \frac{6}{3+j\Omega} \rightarrow \boxed{X^{-1}(j\Omega) = 6 \cdot e^{-3t}\,u(t)}$

g) $X(j\Omega) = 20\,\mathrm{tri}(8\Omega) \rightarrow \boxed{X^{-1}(j\Omega) = \frac{5}{4\pi}\,\mathrm{sinc}^2\left(\frac{t}{16\pi}\right)}$

$\mathrm{tri}(t) \overset{FT}{\longleftrightarrow} \mathrm{sinc}^2\left(\frac{\Omega}{2\pi}\right)$

$\mathrm{sinc}^2\left(\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\pi\,\mathrm{tri}(\Omega)$

$\mathrm{sinc}^2\left(\frac{t}{16\pi}\right) \overset{FT}{\longleftrightarrow} 16\pi\,\mathrm{tri}(8\Omega)$

$\frac{20}{16\pi}\,\mathrm{sinc}^2\left(\frac{t}{16\pi}\right) \overset{FT}{\longleftrightarrow} 20\,\mathrm{tri}(8\Omega)$

h) $X(j\Omega) = 0{,}375\,\mathrm{rect}\left(\frac{\Omega}{16\pi}\right) e^{j7\Omega} \rightarrow \boxed{X^{-1}(j\Omega) = 3\,\mathrm{sinc}(8(t+7))}$

$\mathrm{rect}(t) \overset{FT}{\longleftrightarrow} \mathrm{sinc}\left(\frac{\Omega}{2\pi}\right)$

$\mathrm{sinc}\left(\frac{t}{2\pi}\right) \overset{FT}{\longleftrightarrow} 2\pi\,\mathrm{rect}(\Omega)$

$\mathrm{sinc}\left(\frac{16\pi t}{2\pi}\right) \overset{FT}{\longleftrightarrow} \frac{2\pi}{16\pi}\,\mathrm{rect}\left(\frac{\Omega}{16\pi}\right)$

$\mathrm{sinc}(8t) \overset{FT}{\longleftrightarrow} \frac{1}{8}\,\mathrm{rect}\left(\frac{\Omega}{16\pi}\right) = 0{,}125\,\mathrm{rect}\left(\frac{\Omega}{16\pi}\right)$

$\frac{0{,}375}{0{,}125}\,\mathrm{sinc}(8t) \overset{FT}{\longleftrightarrow} 0{,}375\,\mathrm{rect}\left(\frac{\Omega}{16\pi}\right)$

$3\,\mathrm{sinc}(8t) \overset{FT}{\longleftrightarrow} 0{,}375\,\mathrm{rect}\left(\frac{\Omega}{16\pi}\right)$

$x(t-t_0) \overset{FT}{\longleftrightarrow} X(j\Omega)\,e^{-j\Omega t_0}$

i) $X(j\Omega) = e^{j3\Omega}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j3\Omega} \cdot e^{j\Omega t}\, d\Omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\Omega(3+t)}\, d\Omega = \frac{1}{2\pi}\left[\frac{e^{j\Omega(3+t)}}{j(3+t)}\right]_{-\infty \to 0}^{\infty \to \infty} =$$

$$= \infty$$

j) $X(j\Omega) = 3\,\text{sinc}^2\left(\frac{3\Omega}{\pi}\right)e^{j5\Omega} \longrightarrow \boxed{X^{-1}(j\Omega) = \frac{1}{2}\,\text{tri}\left(\frac{t+5}{6}\right)}$

$$\text{tri}(t) \overset{TF}{\longleftrightarrow} \text{sinc}^2\left(\frac{\Omega}{2\pi}\right)$$

$$\text{tri}\left(t/6\right) \overset{TF}{\longleftrightarrow} 6\,\text{sinc}^2\left(\frac{3\Omega}{\pi}\right)$$

$$\frac{3}{6}\,\text{tri}\left(t/6\right) \overset{TF}{\longleftrightarrow} 3\,\text{sinc}^2\left(\frac{3\Omega}{\pi}\right)$$

k) $X(j\Omega) = 96\,\text{sinc}\left(\frac{4\Omega}{\pi}\right)e^{-j\Omega} \longrightarrow \boxed{X^{-1}(j\Omega) = 12\,\text{rect}\left(\frac{t-1}{8}\right)}$

$$\text{rect}(t) \overset{TF}{\longleftrightarrow} \text{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\text{rect}\left(t/8\right) \overset{TF}{\longleftrightarrow} 8\,\text{sinc}\left(\frac{4\Omega}{\pi}\right)$$

$$\frac{96}{8}\,\text{rect}\left(t/8\right) \overset{TF}{\longleftrightarrow} 96\,\text{sinc}\left(\frac{4\Omega}{\pi}\right)$$

l) $X(j\Omega) = \text{rect}(\Omega + 10\pi) - \text{rect}(\Omega - 10\pi) \longrightarrow \boxed{X^{-1}(j\Omega) = \frac{1}{2\pi}\,\text{sinc}\left(\frac{t}{2\pi}\right)e^{-j10\pi t} - \frac{1}{2\pi}\,\text{sinc}\left(\frac{t}{2\pi}\right)e^{j10\pi t}}$

$$\text{rect}(t) \overset{FT}{\longleftrightarrow} \text{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\text{sinc}\left(t/2\pi\right) \overset{FT}{\longleftrightarrow} 2\pi\,\text{rect}(\Omega)$$

$$\frac{1}{2\pi}\,\text{sinc}\left(t/2\pi\right) \overset{FT}{\longleftrightarrow} \text{rect}(\Omega)$$

Prop. deslocamento em frequência:

$$x(t) \overset{FT}{\longleftrightarrow} X(j\Omega)$$

$$y(t) = x(t)e^{+j\Omega_0 t} \overset{FT}{\longleftrightarrow} Y(j\Omega) = X(j(\Omega - \Omega_0))$$

m) $X(j\Omega) = 48\cos(3\Omega) \rightarrow \boxed{X^{-1}(j\Omega) = 24\left[\delta(t+3) + \delta(t-3)\right]}$

$\cos 3t \overset{TF}{\longleftrightarrow} \pi\left[\delta(\Omega+3) + \delta(\Omega-3)\right]$

$\pi\left[\delta(t+3) + \delta(t-3)\right] \overset{TF}{\longleftrightarrow} 2\pi\cos(3\cdot -\Omega)$
$= 2\pi\cos(3\Omega)$

$\frac{48}{2\pi}\cdot\pi\left[\delta(t+3) + \delta(t-3)\right] \overset{TF}{\longleftrightarrow} 48\cos(3\Omega)$

$24\left[\delta(t+3) + \delta(t-3)\right] \overset{TF}{\longleftrightarrow} 48\cos(3\Omega)$

n) $X(j\Omega) = j\frac{16}{3}\operatorname{sen}(\Omega) \rightarrow \boxed{X^{-1}(j\Omega) = \frac{8}{3}\left[\delta(t+1) - \delta(t-1)\right]}$

$\operatorname{sen} t \overset{TF}{\longleftrightarrow} j\pi\left[\delta(\Omega+1) - \delta(\Omega-1)\right]$

$j\pi\left[\delta(t+1) - \delta(t-1)\right] \overset{TF}{\longleftrightarrow} 2\pi\operatorname{sen}(-\Omega)$
$= -2\pi\operatorname{sen}(\Omega)$

$-j\frac{16/3}{2\pi}\cdot j\pi\left[\delta(t+1) - \delta(t-1)\right] \overset{TF}{\longleftrightarrow} -j\frac{16/3}{\cancel{2\pi}}\cdot -\cancel{2\pi}\operatorname{sen}(\Omega)$

$\frac{8}{3}\left[\delta(t+1) - \delta(t-1)\right] \overset{TF}{\longleftrightarrow} j\frac{16}{3}\operatorname{sen}(\Omega)$

o) $X(j\Omega) = \frac{16}{3}\operatorname{sen}(\Omega) \rightarrow \boxed{X^{-1}(j\Omega) = -j\frac{8}{3}\left[\delta(t+1) - \delta(t-1)\right]}$

$\operatorname{sen} t \overset{TF}{\longleftrightarrow} j\pi\left[\delta(\Omega+1) - \delta(\Omega-1)\right]$

$j\pi\left[\delta(t+1) - \delta(t-1)\right] \overset{TF}{\longleftrightarrow} 2\pi\operatorname{sen}(-\Omega) = -2\pi\operatorname{sen}(\Omega)$

$\underbrace{-\frac{\frac{16}{3}}{2\pi}}\, j\pi\left[\delta(t+1) - \delta(t-1)\right] \overset{TF}{\longleftrightarrow} -\frac{\frac{16}{3}}{\cancel{2\pi}}\cdot -\cancel{2\pi}\operatorname{sen}(\Omega)$

$\Downarrow$

$-\frac{16}{6\pi}\cdot j\pi$

$-j\frac{8}{3}\left[\delta(t+1) - \delta(t-1)\right] \overset{TF}{\longleftrightarrow} \frac{16}{3}\operatorname{sen}(\Omega)$

p) $X(j\Omega) = -j\frac{16}{3}\,\text{sen}(\Omega) \longrightarrow \boxed{X^{-1}(j\Omega) = -\frac{8}{3}\left[\delta(t+1) - \delta(t-1)\right]}$

da letra n

q) $X(j\Omega) = j\frac{16}{3}\cos(\Omega) \rightarrow \boxed{X^{-1}(j\Omega) = j\frac{8}{3}\left[\delta(t+1) + \delta(t-1)\right]}$

$$\cos t \overset{TF}{\longleftrightarrow} \pi\left[\delta(\Omega+1) + \delta(\Omega-1)\right]$$

$$\pi\left[\delta(t+1) + \delta(t-1)\right] \overset{TF}{\longleftrightarrow} 2\pi\cos(\Omega)$$

$$\underbrace{\frac{j\frac{16}{3}}{2\pi}}\cdot\pi\left[\delta(t+1) + \delta(t-1)\right] \overset{TF}{\longleftrightarrow} j\frac{16}{3}\cos(\Omega)$$

$$\underbrace{j\frac{16}{6\cancel{\pi}}\cdot\cancel{\pi}}_{j\,8/3}$$

Exercícios de sistemas

1) $h_1(t) = 3e^{-10t}u(t)$

$h_2(t) = \delta(t) - 3e^{-10t}u(t)$

Em cascata:

$h(t) = h_1(t) * h_2(t) \longrightarrow$ OK!

$H(j\Omega) = H_1(j\Omega)H_2(j\Omega)$

$H_1(j\Omega) = \dfrac{3}{10+j\Omega}$ $\qquad H_2(j\Omega) = 1 - \dfrac{3}{10+j\Omega}$

$$H(j\Omega) = \left[\frac{3}{10+j\Omega}\right]\left[1 - \frac{3}{10+j\Omega}\right]$$

$$H(j\Omega) = \frac{3}{10+j\Omega} - \frac{9}{(10+j\Omega)^2}$$

Em paralelo:

$h(t) = h_1(t) + h_2(t) = \delta(t)$

$\boxed{H(j\Omega) = 1}$

2)

$$h(t) = 10\,\mathrm{rect}\left(\frac{t - 0{,}01}{0{,}02}\right)$$

$$10\,\mathrm{rect}(t) \overset{FT}{\longleftrightarrow} 10\,\mathrm{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$10\,\mathrm{rect}\left(\frac{t}{0{,}02}\right) \overset{FT}{\longleftrightarrow} 0{,}2\,\mathrm{sinc}\left(\frac{0{,}01\,\Omega}{\pi}\right)$$

$$10\,\mathrm{rect}\left(\frac{t - 0{,}01}{0{,}02}\right) \overset{FT}{\longleftrightarrow} 0{,}2\,\mathrm{sinc}\left(\frac{0{,}01\,\Omega}{\pi}\right) e^{-j\Omega \cdot 0{,}01}$$

$$|H(j\Omega)| = 0{,}2 \left|\mathrm{sinc}\left(\frac{0{,}01\,\Omega}{\pi}\right)\right|$$

$$\mathrm{sinc}\left(\frac{0{,}01\,\Omega}{\pi}\right) = \frac{\mathrm{sen}\,(0{,}01\,\Omega)}{0{,}01\,\Omega}$$

→ primeiro zero à esquerda

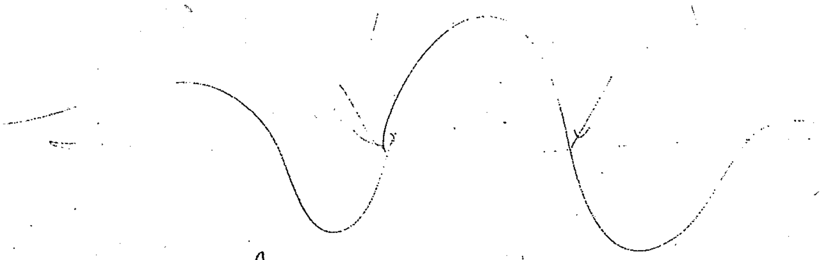
$0{,}01\,\Omega = -\pi$

$\Omega = -100\pi$

→ primeiro zero à direita

$0{,}01\,\Omega = \pi$

$\Omega = 100\pi$



Largura de banda nula é $100\pi - (-100\pi) = 200\pi$

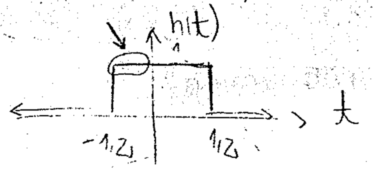
3)

a) causal: $h(t) = 0 \,, \ t < 0$

não causal: $h(t) \neq 0 \,, \ t < 0$

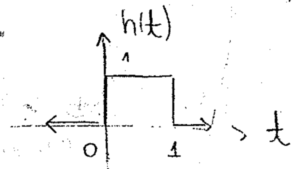
$$H(j\Omega) = \operatorname{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$h(t) = \operatorname{rect}(t) \Rightarrow \text{não causal, } h(t) \neq 0, \ t < 0$$



b) $$H(j\Omega) = \operatorname{sinc}\left(\frac{\Omega}{2\pi}\right) e^{-j\Omega/2}$$

$$h(t) = \operatorname{rect}(t - 1/2) \Rightarrow \text{causal, } h(t) = 0, \ t < 0$$

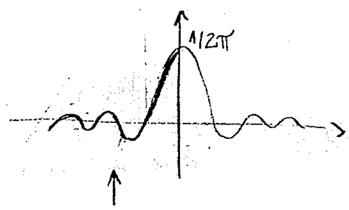


c) $$H(j\Omega) = \operatorname{rect}(\Omega) \longrightarrow h(t) = \frac{1}{2\pi} \operatorname{sinc}\left(\frac{t}{2\pi}\right) \Rightarrow \text{não causal, } h(t) \neq 0; \ t < 0$$

$$\operatorname{rect}(t) \overset{TF}{\longleftrightarrow} \operatorname{sinc}\left(\frac{\Omega}{2\pi}\right)$$

$$\operatorname{sinc}\left(\frac{t}{2\pi}\right) \overset{TF}{\longleftrightarrow} 2\pi \operatorname{rect}(\Omega)$$

$$\frac{1}{2\pi} \operatorname{sinc}\left(\frac{t}{2\pi}\right) \overset{TF}{\longleftrightarrow} \operatorname{rect}(\Omega)$$

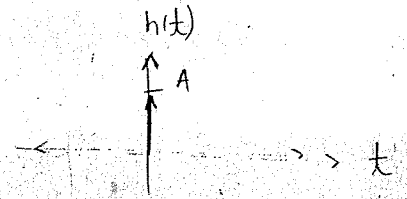


d) $$H(j\Omega) = \operatorname{rect}(\Omega)\, e^{-j\Omega} \rightarrow h(t) = \frac{1}{2\pi} \operatorname{sinc}\left(\frac{t-1}{2\pi}\right)$$

$$\Rightarrow \text{não causal, } h(t) \neq 0, \ t < 0$$

e) $H(j\Omega) = A$

$h(t) = A\,\delta(t)$



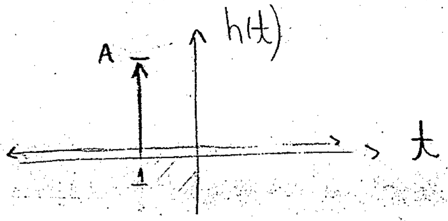
$h(t) = 0,\ t \neq 0$, Pelo gráfico

$\therefore\ h(t) = 0,\ t < 0 \Rightarrow$ causal

f) $H(j\Omega) = A e^{j\Omega}$

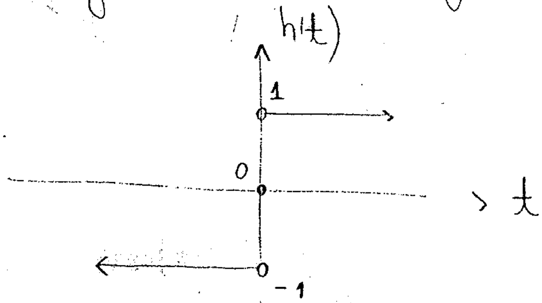
Pelo gráfico,

$h(t) = A\,\delta(t+1) \Rightarrow h(t) \neq 0,\ t < 0 \Rightarrow$ não causal



g) $H(j\Omega) = \dfrac{2}{j\Omega} \rightarrow h(t) = sgn(t) \Rightarrow$ Pelo gráfico,

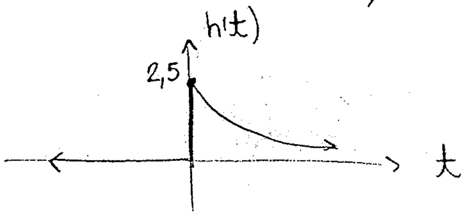
$h(t) \neq 0,\ t < 0 \Rightarrow$ não causal



h) $H(j\Omega) = \dfrac{10}{6 + j4\Omega} = \dfrac{10/4}{6/4 + j\Omega} = \dfrac{2{,}5}{1{,}5 + j\Omega}$

$h(t) = 2{,}5\, e^{-1{,}5t}\, u(t) \Rightarrow$ Pelo gráfico,

$h(t) = 0,\ t < 0 \Rightarrow$ causal



i) $H(j\Omega) = \dfrac{4}{25 + (j\Omega)^2 + 6(j\Omega)} = \dfrac{4}{(j\Omega)^2 + 6(j\Omega) + 25} =$

$x^2 + 6x + 25$

$\begin{cases} -3 + j4 \\ -3 - j4 \end{cases}$

$= \dfrac{4}{[j\Omega - (-3+j4)][j\Omega - (-3-j4)]} =$

$= \dfrac{4}{[3 - j4 + j\Omega][3 + j4 + j\Omega]} = \dfrac{A}{3 - j4 + j\Omega} + \dfrac{B}{3 + j4 + j\Omega}$

$4 = [3 + j4 + j\Omega]A + [3 - j4 + j\Omega]B$

* se $j\Omega = -3 - j4$

$4 = [\cancel{3} - j4 \cancel{-3} - j4]B$

$B = \dfrac{4}{-j8} \cdot \dfrac{j}{j} = \dfrac{4j}{8} = j/2$

* se $j\Omega = -3 + j4$:

$4 = [\cancel{3} + j4 \cancel{-3} + j4]A$

$A = \dfrac{4}{j8} \cdot \dfrac{j}{j} = \dfrac{4j}{-8} = -j/2$

$H(j\Omega) = \dfrac{-j/2}{[\underbrace{3 - j4}_{a} + j\Omega]} + \dfrac{j/2}{[\underbrace{3 + j4}_{a} + j\Omega]}$

$h(t) = -\dfrac{j}{2} e^{-(3-j4)t} u(t) + \dfrac{j}{2} e^{-(3+j4)t} u(t)$

$h(t) = 0,\ t < 0 \Rightarrow$ causal

j) $h(t) = -\frac{j}{2} e^{-(3-j4)(t+1)} u(t+1) + \frac{j}{2} e^{-(3+j4)(t+1)} u(t+1)$

$h(t) \neq 0 \ , \ t < 0 \Rightarrow$ não causal

k) $h(t) = -\frac{j}{2} e^{-(3-j4)(t-1)} u(t-1) + \frac{j}{2} e^{-(3-j4)(t-1)} u(t-1)$

$h(t) = 0, \ t < 0 \Rightarrow$ causal

l) $H(j\Omega) = \frac{9 + j\Omega}{(j\Omega)^2 + 6(j\Omega) + 45} = \frac{9 + j\Omega}{[j\Omega - (-3+j6)][j\Omega - (-3-j6)]}$

$= \frac{A}{[j\Omega - (-3+j6)]} + \frac{B}{[j\Omega - (-3-j6)]}$

$9 + j\Omega = [j\Omega + 3 + j6] A + [j\Omega + 3 - j6] B$

$9 + (-3 - j6) = (\cancel{-3} - j6 \cancel{+3} - j6) B$

$6 - j6 = -j12 B \rightarrow B = \frac{6 - j6}{-j12} \cdot \frac{j}{j} = \frac{6j + 6}{12} = 0{,}5j + 0{,}5$

$9 - 3 + j6 = \cancel{-3} + j6 \cancel{+3} + j6 \ A$

$A = \frac{6 + j6}{j12} \cdot \frac{j}{j} = \frac{6j - 6}{-12} = -0{,}5j + 0{,}5$

$H(j\Omega) = \frac{-0{,}5j + 0{,}5}{[3 - j6 + j\Omega]} + \frac{0{,}5j + 0{,}5}{[3 + j6 + j\Omega]}$

$h(t) = (-0{,}5j + 0{,}5) e^{-(3-j6)t} u(t) + (0{,}5j + 0{,}5) e^{-(3+j6)t} u(t)$

$h(t) = 0, \ t < 0 \Rightarrow$ causal

m) $H(j\Omega) = \dfrac{49}{49+\Omega^2} = \dfrac{49}{49-(j\Omega)^2} = \dfrac{49}{(j\Omega-7)(j\Omega+7)} = \dfrac{A}{j\Omega-7} + \dfrac{B}{j\Omega+7}$

$49-(j\Omega)^2=0$

$(j\Omega)^2 = 49$

$(j\Omega) = \pm 7$

$49 = (j\Omega+7)A + (j\Omega-7)B$

$49 = -14B \Rightarrow B = -3{,}5$

$49 = 14A \Rightarrow A = 3{,}5$

$H(j\Omega) = \dfrac{3{,}5}{j\Omega-7} - \dfrac{3{,}5}{j\Omega+7} \rightarrow h(t) = 3{,}5\,e^{7t}u(t) - 3{,}5\,e^{-7t}u(t)$

$h(t) = 0,\ t<0 \Rightarrow$ causal

4)

c) $$V_{out}(s) = \frac{R_1}{R_1 + \left(\frac{sL_1 \cdot \frac{1}{sC_1}}{sL_1 + \frac{1}{sC_1}}\right)} V_{in}(s)$$

$$H(s) = \frac{R_1}{R_1 + \left(\frac{\frac{sL_1}{\cancel{sC_1}}}{\frac{s^2 L_1C_1 + 1}{\cancel{sC_1}}}\right)} = \frac{R_1}{R_1 + \left(\frac{sL_1}{s^2 L_1 C_1 + 1}\right)} =$$

$$H(j\Omega) = \frac{(j\Omega)^2 + 10^9}{(j\Omega)^2 + 10^3 j\Omega + 10^9} = \frac{(j\Omega - j31623)(j\Omega + j31623)}{(j\Omega + 500 - j31619)(j\Omega + 500 + j31619)} =$$

$$= \frac{(-j31623)\left(1 - \frac{j\Omega}{-j31623}\right)\left(1 + \frac{j\Omega}{j31623}\right)(j31623)}{(500 - j31619)\left(1 + \frac{j\Omega}{500 - j31619}\right)\left(\frac{1 + j\Omega}{500 + j31619}\right)(500 + j31619)}$$

$(j\Omega)^2 + 10^9 \rightarrow \pm j31623$

$(j\Omega)^2 + 10^3 j\Omega + 10^9 \left< \begin{array}{l} -500 + j31619 \\ -500 - j31619 \end{array} \right.$

$$H(j\Omega) = 1 \cdot \frac{\left(1 + \frac{j\Omega}{-j31623}\right)\left(1 + \frac{j\Omega}{j31623}\right)}{\left(1 + \frac{j\Omega}{500 - j31619}\right)\left(1 + \frac{j\Omega}{500 + j31619}\right)}$$

D) $$I_2 = \frac{sL_1}{sL_1 + R_1 + \frac{1}{sC_1}} \cdot I_{in}$$

$$V_{out} = R_1 I_2$$
$$V_{out} = \frac{sL_1}{sL_1 + R_1 + \frac{1}{sC_1}} I_{in} \cdot R_1$$

$$H(s) = \frac{sL_1 R_1}{sL_1 + R_1 + \frac{1}{sC_1}} \rightarrow H(j\Omega) = \frac{100\,(j\Omega)^2}{(j\Omega)^2 + 10^5 (j\Omega) + 10^9}$$

$$j\Omega^2 + 10^5 (j\Omega) + 10^9 \quad \begin{cases} -11270,17 \\ -88729,83 \end{cases}$$

$$H(j\Omega) = 100 \frac{j\Omega \cdot j\Omega}{\frac{11270,17}{11270,17} (j\Omega + 11270,17)(j\Omega + 88729,83) \cdot \frac{88729,83}{88729,83}}$$

$$H(j\Omega) = 10 \cdot 10^{-8} \frac{j\Omega \cdot j\Omega}{\left(1 + \frac{j\Omega}{11270,17}\right)\left(1 + \frac{j\Omega}{88729,83}\right)}$$

f) $V_{out}(s) = \dfrac{R_1}{R_1 + sL_1 + \dfrac{1}{sC_1}} V_{in}(s)$ $\quad R_1 = 1k\ ;\ C_1 = 1\mu F,\ L_1 = 1mH$

$$H(s) = \frac{10^6 s}{s^2 + 10^6 s + 10^9} \rightarrow H(j\Omega) = \frac{10^6 (j\Omega)}{(j\Omega)^2 + 10^6 (j\Omega) + 10^9}$$

$$(j\Omega)^2 + 10^6 (j\Omega) + 10^9 \rightarrow \begin{array}{l} -1000 \\ -10^6 \end{array}$$

i) $R_1 = 1K \quad C_1 = 1\mu F \quad L_1 = 1mH$

) $V_{out}(s) = \dfrac{sL_1 + \frac{1}{sC_1}}{sL_1 + \frac{1}{sC_1} + R_1} \cdot V_{in}(s)$

$$H(s) = \frac{\frac{s^2L_1C_1 + 1}{\cancel{sC_1}}}{\frac{s^2L_1C_1 + 1 + sZ_1C_1}{\cancel{sC_1}}} = \frac{10^{-9}s^2 + 1}{10^{-9}s^2 + 1 + 10^{-3}s}$$

$$H(j\Omega) = \frac{10^{-9}(j\Omega)^2 + 1}{10^{-9}(j\Omega)^2 + 10^{-3}(j\Omega) + 1} = \frac{(j\Omega)^2 + 10^9}{(j\Omega)^2 + 10^6(j\Omega) + 10^9}$$

$(j\Omega)^2 + 10^9 \rightarrow \pm j31623$

$(j\Omega)^2 + 10^6(j\Omega) + 10^9 \rightarrow -1000$

-10^6

6) $R_1 = 1K, \; R_2 = 100, \; C_1 = 40 \, mF$

c) Κόμβος 1:

$$\frac{0 - X(s)}{R_1 + \frac{1}{sC_1}} + \frac{0 - Y(s)}{R_2} = 0$$

$$\frac{-X(s)}{R_1 + \frac{1}{sC_1}} - \frac{Y(s)}{R_2} = 0 \;\rightarrow\; \frac{X(s)}{R_1 + \frac{1}{sC_1}} + \frac{Y(s)}{R_2} = 0$$

$$\frac{Y(s)}{R_2} = -\frac{X(s)}{R_1 + \frac{1}{sC_1}} \;\rightarrow\; \frac{Y(s)}{X(s)} = -\frac{R_2}{R_1 + \frac{1}{sC_1}}$$

$$H(s) = -\frac{4s}{40s + 1} \;\longrightarrow\; H(j\Omega) = \frac{-4(j\Omega)}{40(j\Omega) + 1} \quad .40/40$$

$$H(j\Omega) = \frac{-4\left(j\frac{\Omega}{1}\right)}{40\left(j\Omega + 1/40\right)} = -\frac{4}{40} \, \frac{j\frac{\Omega}{1}}{\left(j\Omega + 1/40\right) \cdot \frac{1/40}{1/40}} =$$

$$= -\frac{4}{\cancel{40} \cdot 1/\cancel{40}} \, \frac{j\frac{\Omega}{1}}{\left(1 + j\frac{\Omega}{0{,}025}\right)} = -4 \cdot \frac{j\frac{\Omega}{1}}{\left(1 + j\frac{\Omega}{0{,}025}\right)}$$

d) Νό 1: $R_2 = 100\,\Omega,\ R_1 = 1K,\ C_1 = 40\,mF$

$$\frac{0 - X(s)}{R_1} + \frac{0 - Y(s)}{\dfrac{R_2 \cdot 1/sC_1}{R_2 + \dfrac{1}{sC_1}}} = 0$$

$$\frac{X(s)}{R_1} + \frac{Y(s)}{\dfrac{\frac{R_2}{sC_1}}{R_2 + \frac{1}{sC_1}}} = 0 \rightarrow \frac{Y(s)}{\dfrac{\frac{R_2}{sC_1}}{R_2 + \frac{1}{sC_1}}} = -\frac{X(s)}{R_1}$$

$$H(s) = -\frac{\dfrac{\frac{R_2}{sC_1}}{R_2 + 1/sC_1}}{R_1} = -\frac{1}{40s + 10}$$

$$H(j\Omega) = -\frac{1}{(40(j\Omega) + 10)}\ \frac{40}{40} = -\frac{1}{40\,(j\Omega + 0{,}25)} \cdot \frac{0{,}25}{0{,}25}$$

$$= -\frac{1}{40 \cdot 0{,}25}\ \frac{1}{\left(1 + \frac{j\Omega}{0{,}25}\right)} = \frac{-0{,}1}{\left(1 + \frac{j\Omega}{0{,}25}\right)}$$

7)

B) Νό 2:

$$\frac{Y(s) - V_1}{\frac{1}{sC_2}} + \frac{Y(s)}{R_2} = 0$$

$$Y(s) \cdot sC_2 - sC_2 V_1 + \frac{Y(s)}{R_2} = 0$$

$$Y(s) \left(sC_2 + \frac{1}{R_2} \right) = sC_2 V_1$$

$$V_1 = Y(s) \cdot \frac{\left(sC_2 + \frac{1}{R_2} \right)}{sC_2} = \frac{s+100}{s} Y(s)$$

Νό 1

$$\frac{V_1 - X(s)}{\frac{1}{sC_1}} + \frac{V_1}{R_1} + \frac{V_1 - Y(s)}{1/sC_2} = 0$$

$$sC_1 V_1 - X(s) sC_1 + \frac{V_1}{R_1} + sC_2 V_1 - Y(s) \cdot sC_2 = 0$$

$$V_1 \left(sC_1 + \frac{1}{R_1} + sC_2 \right) - Y(s) \cdot sC_2 = X(s) \cdot sC_1$$

$$\left(\frac{s+100}{s} \right) Y(s) \left(sC_1 + \frac{1}{R_1} + sC_2 \right) - Y(s) \cdot sC_2 = X(s) \cdot sC_1$$

$$Y(s) \left[\left(\frac{s+100}{s} \right) \left(sC_1 + \frac{1}{R_1} + sC_2 \right) - sC_2 \right] = X(s)\, sC_1$$

$$H(s) = \frac{s^2}{s^2 + 300s + 10000} \rightarrow H(j\Omega) = \frac{(j\Omega)^2}{(j\Omega)^2 + 300(j\Omega) + 10000}$$

$$(j\Omega)^2 + 300(j\Omega) + 10000 \quad \begin{matrix} \nearrow -38,2 \\ \searrow -261,8 \end{matrix}$$

c) Νό 2:

$$\frac{Y(s)-X(s)}{R_3}+\frac{Y(s)-V_1}{\frac{1}{sC_2}}+\frac{Y(s)}{R_2}=0$$

$$\frac{Y(s)}{R_3}-\frac{X(s)}{R_3}+sC_2Y(s)-sC_2V_1+\frac{Y(s)}{R_2}=0$$

$$Y(s)\left(\frac{1}{R_3}+\frac{1}{R_2}+sC_2\right)-\frac{X(s)}{R_3}=sC_2V_1$$

$$V_1=\frac{Y(s)\left(\frac{1}{R_3}+\frac{1}{R_2}+sC_2\right)}{sC_2}-\frac{X(s)}{sC_2R_3}=\frac{s+1100}{s}Y(s)-\frac{100}{s}X(s)$$

$$V_1=\frac{s+1100}{s}Y(s)-\frac{100}{s}X(s)$$

Νό 1:

$$\frac{V_1-X(s)}{1/sC_1}+\frac{V_1}{R_1}+\frac{V_1-Y(s)}{1/sC_2}=0$$

$$sC_1V_1-sC_1X(s)+\frac{V_1}{R_1}+sC_2V_1-sC_2Y(s)=0$$

$$sC_1\left(\frac{s+1100}{s}\right)Y(s)-sC_1\frac{100}{s}X(s)-sC_1X(s)+\frac{1}{R_1}\left(\frac{s+1100}{s}\right)Y(s)-\frac{1}{R_1}\frac{100}{s}X(s)$$

$$+\,sC_2\left(\frac{s+1100}{s}\right)Y(s)-sC_2\frac{100}{s}X(s)-sC_2Y(s)=0$$

$$Y(s)\left[C_1(s+1100)+\frac{1}{R_1}\left(\frac{s+1100}{s}\right)+C_2(s+1100)-sC_2\right]=$$

$$=X(s)\left[100C_1+sC_1+\frac{100}{R_1s}+100C_2\right]$$

$$Y(s)\left[\frac{s^2+2300s+110000}{10^6s}\right]=X(s)\left[\frac{s^2+200s+10^4}{10^6s}\right]$$

$$H(s)=\frac{s^2+200s+10^4}{s^2+2300s+110000}\Rightarrow H(j\Omega)=\frac{(j\Omega)^2+200j\Omega+10^4}{(j\Omega)^2+2300(j\Omega)+110000}$$

$$(j\Omega)^2+200j\Omega+10^4 \Big< \begin{matrix}-100\\-100\end{matrix}$$

$$(j\Omega)^2+2300(j\Omega)+110000 \Big< \begin{matrix}-48{,}864\\-2251{,}136\end{matrix}$$

$$H(j\Omega)=\frac{(j\Omega+100)\cdot(j\Omega+100)}{(j\Omega+48{,}864)(j\Omega+2251{,}136)}$$

8) Κόμβος 2:

$R_2 = 1K \quad R_3 = 10K$
$R_1 = 10K \quad C_1 = 1\mu F$
$C_2 = 1\mu F$

$$\frac{0 - Y(s)}{\frac{1}{sC_1}} + \frac{0 - V_1}{R_2} = 0$$

$$sC_1 Y(s) + \frac{V_1}{R_2} = 0 \rightarrow \frac{V_1}{R_2} = -sC_1 Y(s)$$

$$V_1 = -s R_2 C_1 Y(s)$$

$$V_1 = -10^{-3} s\, Y(s)$$

Κόμβος 1:

$$\frac{V_1 - X(s)}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - Y(s)}{R_3} + \frac{V_1}{\frac{1}{sC_2}} = 0$$

$$\frac{V_1}{R_1} - \frac{X(s)}{R_1} + \frac{V_1}{R_2} + \frac{V_1}{R_3} - \frac{Y(s)}{R_3} + sC_2 V_1 = 0$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + sC_2\right) - \frac{Y(s)}{R_3} = \frac{X(s)}{R_1}$$

$$-10^{-3} s\, Y(s)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + sC_2\right) - \frac{Y(s)}{R_3} = \frac{X(s)}{R_1}$$

$$Y(s)\left[-10^{-3} s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + sC_2\right) - \frac{1}{R_3}\right] = \frac{X(s)}{R_1}$$

$$Y(s)\left[-\left(\frac{s^2 + 1200s + 10^5}{10^9}\right)\right] = \frac{X(s)}{10^4}$$

$$H(s) = \frac{Y(s)}{X(s)} = -\frac{10^9}{10^4(s^2 + 1200s + 10^5)} = \frac{-10^5}{s^2 + 1200s + 10^5}$$

$$H(j\Omega) = \frac{-10^5}{(j\Omega)^2 + 1200(j\Omega) + 10^5} = \frac{-10^5}{(j\Omega + 90)(j\Omega + 1110)}$$

$$(j\Omega)^2 + 1200(j\Omega) + 10^5 < \begin{matrix} -90 \\ -1110 \end{matrix}$$

a 3 - Sinais e Sistemas

Parte 1 - Transformada de Laplace

1)

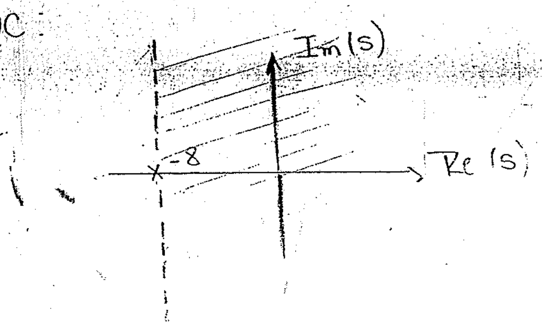
a) $x(t) = e^{-8t} u(t)$

$$X(s) = \int_0^{\infty} e^{-8t} \cdot e^{-st} \, dt = \int_0^{\infty} e^{-(8+s)t} \, dt =$$

$\mathrm{Re}\{s\} + 8 > 0$

$$= \left. \frac{e^{-(8+s)t}}{-(8+s)} \right|_0^{\infty} = \frac{e^{-(8+s)\cdot\infty} - 1}{-(8+s)} = \frac{1}{s+8}, \quad \mathrm{Re}\{s\} > -8$$

ROC:



b) $x(t) = e^{3t} \cos(20\pi t) u(t) \rightarrow X(s) = \int_0^{\infty} e^{3t} \left[\frac{e^{j20\pi t} + e^{-j20\pi t}}{2} \right] e^{-st} \, dt$

$e^{-(s-3)t} \rightarrow \mathrm{Re}\{s\} - 3 > 0$

$\mathrm{Re}\{s\} > 3$

$$X(s) = \frac{(s-3)}{(s-3)^2 + (20\pi)^2}$$

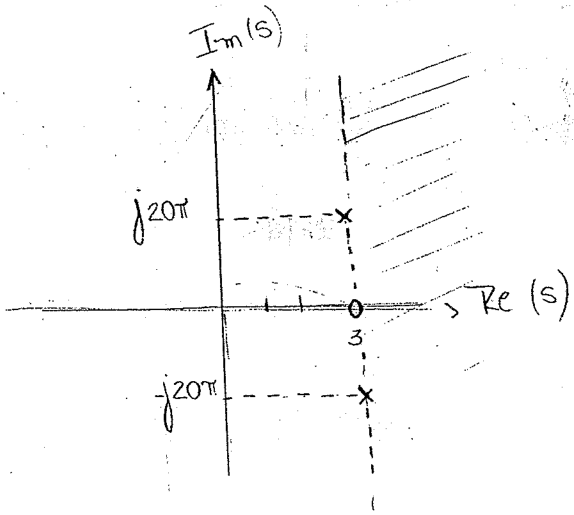
ROC:

$(s-3)^2 + (20\pi)^2 = 0$

$(s-3)^2 = -(20\pi)^2$

$(s-3) = \pm j20\pi$

$s = 3 \pm j20\pi$



causal

não estável

i) $x(t) = e^{2t}u(-t) - 5e^{-5t}u(t)$

$$X(s) = -\frac{1}{s-2} - \frac{5}{s+5} \qquad \frac{-6s+5}{s^2+3s-10}$$

ROC: $Re\{s\} < 2$, $Re\{s\} > -5$

$$\int_{-\infty}^{0} e^{2t} \cdot e^{-st} dt =$$

$$= \int_{-\infty}^{0} e^{-(s-2)t} dt = \left. \frac{e^{-(s-2)t}}{-(s-2)} \right|_{-\infty}^{0} =$$

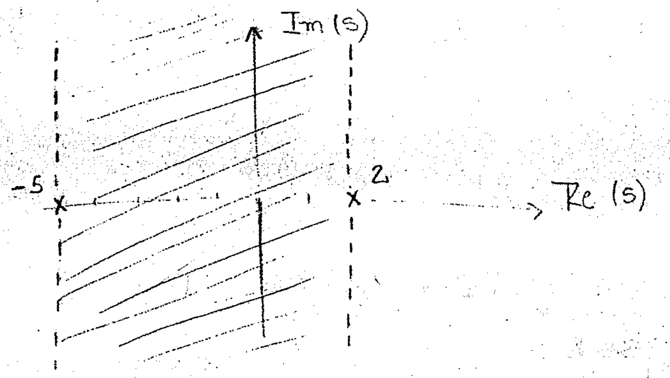
$$= \frac{1 - e^{-(s-2)\cdot -\infty}}{-(s-2)} = \frac{-1}{(s-2)}, \quad Re\{s\} - 2 < 0$$

$$Re\{s\} < 2$$

Raciocínio

$s - 2 < 0$ $\qquad s + 5 > 0$

$s < 2$ $\qquad s > -5$



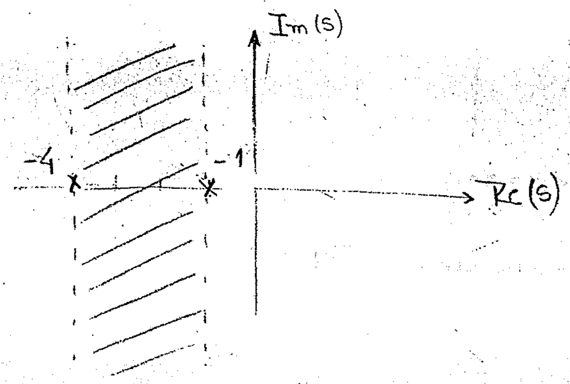
não é causal

nem não - causal

D) $x(t) = e^{-t}u(-t) - 4e^{-4t}u(t)$

$$X(s) = -\frac{1}{s+1} - \frac{4}{s+4}$$

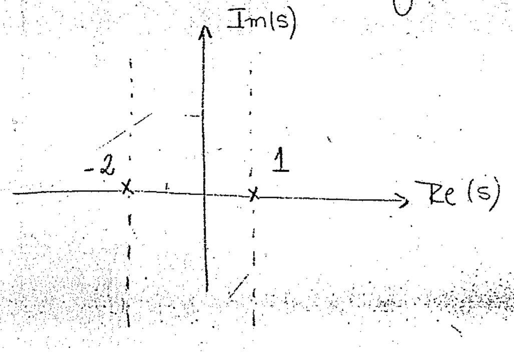
ROC: $Re\{s\} < -1$, $Re\{s\} > -4$



e) $x(t) = e^{-2t}u(-t) - e^{t}u(t)$

$$X(s) = -\frac{1}{s+2} - \frac{1}{s-1}$$

$$\text{ROC: } \underbrace{Re\{s\} < -2 \; ; \; Re\{s\} > 1}_{\text{não há região de convergência}}$$



2)

a) $x(t) = e^{t}u(t)$

$$X(s) = \int_0^{\infty} e^{t} e^{-st} dt = \int_0^{\infty} e^{-(s-1)t} dt = \left. \frac{e^{-(s-1)t}}{-(s-1)} \right|_0^{\infty} =$$

$$= \frac{e^{-(s-1)\infty} - 1}{-(s-1)} = \frac{1}{s-1} \; , \; \boxed{X(s) = \frac{1}{s-1} \, , \; Re\{s\} > 1}$$

b) $x(t) = e^{2t}\cos(200\pi t)u(t)$

$$X(s) = \int_0^{\infty} e^{2t} \left[\frac{e^{j200\pi t} + e^{-j200\pi t}}{2} \right] e^{-st} dt =$$

$$= \frac{1}{2} \left[\int_0^{\infty} e^{2t} e^{j200\pi t} e^{-st} dt + \int_0^{\infty} e^{2t} e^{-st} e^{-j200\pi t} dt \right]$$

$$= \frac{1}{2} \left[\int_0^{\infty} e^{-(s-2-j200\pi)t} dt + \int_0^{\infty} e^{-(s-2+j200\pi)t} dt \right]$$

$$= \frac{1}{2}\left[\frac{e^{-(s-2-j200\pi)t}}{-(s-2-j200\pi)} + \frac{e^{-(s-2+j200\pi)t}}{-(s-2+j200\pi)}\right]_0^{\infty} =$$

$$= \frac{1}{2}\left[\frac{1}{(s-2-j200\pi)} + \frac{1}{(s-2+j200\pi)}\right] = \frac{1}{2}\left[\frac{(s-2+\cancel{j200\pi}) + (s-2-\cancel{j200\pi})}{(s-2)^2-(j200\pi)^2}\right] =$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}(s-2)}{(s-2)^2+(200\pi)^2}\right]$$

$$X(s) = \frac{(s-2)}{(s-2)^2+(200\pi)^2} \qquad \mathrm{Re}\{s\} > 2$$

:) $x(t) = \text{rampa}(t) = t\,u(t)$

$$X(s) = \int_0^{\infty} t\, e^{-st}\,dt = \left[-\frac{t}{s}e^{-st}\Big|_0^{\infty} - \frac{1}{s^2}e^{-st}\Big|_0^{\infty}\right] =$$

$$\begin{array}{ll} t & \oplus\ e^{-st} \\ 1 & \ominus\ -\frac{1}{s}e^{-st} \\ 0 & \frac{1}{s^2}e^{-st} \end{array}$$

$$= \frac{1}{s^2} \Rightarrow X(s) = \frac{1}{s^2} \qquad \mathrm{Re}\{s\} > 0$$

D) $x(t) = t\,e^{t}u(t)$

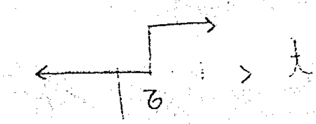
$$X(s) = \int_0^{\infty} t\,e^{t}e^{-st}\,dt = \int_0^{\infty} t\, e^{-(s-1)t}\,dt =$$

$$\begin{array}{ll} t & \oplus\ e^{-(s-1)t} \\ 1 & \ominus\ \frac{e^{-(s-1)t}}{-(s-1)} \\ 0 & \frac{e^{-(s-1)t}}{(s-1)^2} \end{array}$$

$$= \left[-\frac{t}{(s-1)}e^{-(s-1)t}\Big|_0^{\infty} - \frac{e^{-(s-1)t}}{(s-1)^2}\Big|_0^{\infty}\right] =$$

$$= \frac{1}{(s-1)^2} \Rightarrow \boxed{X(s) = \frac{1}{(s-1)^2}} \qquad \mathrm{Re}\{s\} > 1$$

e) $x(t) = e^{-a(t-\tau)}\, u(t-\tau)$



$t-\tau = 0$
$t = \tau$

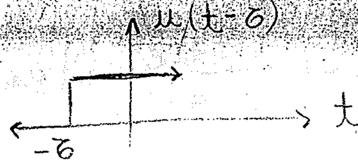
$$X(s) = \int_{-\infty}^{\infty} e^{-a(t-\tau)}\, u(t-\tau)\, e^{-st}\, dt =$$

$$= \int_{\tau}^{\infty} e^{-a(t-\tau)}\, e^{-st}\, dt = \int_{\tau}^{\infty} e^{-at} . e^{a\tau} . e^{-st}\, dt =$$

$$= e^{a\tau} \int_{\tau}^{\infty} e^{-(s+a)t}\, dt = e^{a\tau} \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_{\tau}^{\infty} =$$

$$= e^{a\tau} \left[\frac{e^{-(s+a)\infty} - e^{-(s+a)\tau}}{-(s+a)} \right] = \frac{e^{a\tau} . e^{-(s+a)\tau}}{(s+a)} \nearrow \frac{e^{-s\tau}}{(s+a)} \quad , \ \mathrm{Re}\{s\} > -a$$

f) $x(t) = e^{-a(t+\tau)}\, u(t+\tau)$



$t+\tau = 0$
$t = -\tau$

$$X(s) = \int_{-\infty}^{\infty} e^{-a(t+\tau)}\, u(t+\tau)\, e^{-st}\, dt$$

$$= \int_{-\tau}^{\infty} e^{-a(t+\tau)}\, e^{-st}\, dt = \int_{-\tau}^{\infty} e^{-at} . e^{-a\tau}\, e^{-st}\, dt =$$

$$= e^{-a\tau} \int_{-\tau}^{\infty} e^{-(s+a)t}\, dt = e^{-a\tau} \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_{-\tau}^{\infty} =$$

$$= e^{-a\tau} \left[\frac{e^{-(s+a)\infty} - e^{-(s+a)-\tau}}{-(s+a)} \right] = \frac{e^{-a\tau} . e^{(s+a)\tau}}{(s+a)} = \frac{e^{s\tau}}{(s+a)} \quad , \ \mathrm{Re}\{s\} > -a$$

g) $x(t) = \mathrm{sen}(\Omega_0 t)\, u(t)$

$$X(s) = \int_{0}^{\infty} \mathrm{sen}(\Omega_0 t)\, e^{-st}\, dt = \int_{0}^{\infty} \left[\frac{e^{j\Omega_0 t} - e^{-j\Omega_0 t}}{2j} \right] e^{-st}\, dt =$$

$$= \frac{1}{2j} \left[\int_{0}^{\infty} e^{-(s-j\Omega_0)t}\, dt - \int_{0}^{\infty} e^{-(s+j\Omega_0)t}\, dt \right] =$$

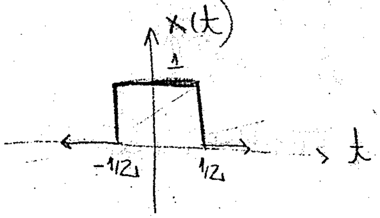
$$= \frac{1}{2j} \left[\frac{e^{-(s-j\Omega_0)t}}{-(s-j\Omega_0)} \Bigg|_{0}^{\infty} + \frac{e^{-(s+j\Omega_0)t}}{(s+j\Omega_0)} \Bigg|_{0}^{\infty} \right] =$$

$$= \frac{1}{2j} \left[\frac{e^{-(s-j\Omega_0)\infty} - 1}{-(s-j\Omega_0)} + \frac{e^{-(s+j\Omega_0)\infty} - 1}{(s+j\Omega_0)} \right] = \frac{1}{2j} \left[\frac{1}{(s-j\Omega_0)} - \frac{1}{(s+j\Omega_0)} \right]$$

$$X(s) = \frac{1}{2j}\left[\frac{\cancel{s}+j\Omega_0 - \cancel{s}+j\Omega_0}{(s^2+\Omega_0^2)}\right] = \frac{\cancel{2j}\,\Omega_0}{\cancel{2j}(s^2+\Omega_0^2)} = \frac{\Omega_0}{s^2+\Omega_0^2}, \quad \mathrm{Re}\{s\} > 0$$

$$s^2 - (j\Omega_0)^2$$
$$s^2 + \Omega_0^2$$

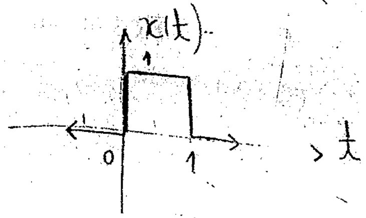
h) $x(t) = \mathrm{rect}(t)$



$$X(s) = \int_{-1/2}^{1/2} 1 \cdot e^{-st}\,dt = -\frac{1}{s}e^{-st}\Big|_{-1/2}^{1/2} = -\frac{1}{s}\left[e^{-s/2} - e^{s/2}\right] =$$

$$= \frac{(e^{s/2} - e^{-s/2})}{s} \quad \Big| \text{ todos os } s$$

i) $x(t) = \mathrm{rect}(t - 1/2)$



$t - 1/2 = -1/2$
$t = 0$
$t - 1/2 = 1/2$
$t = 1$

$$X(s) = \int_0^1 1 \cdot e^{-st}\,dt = -\frac{1}{s}e^{-st}\Big|_0^1 = -\frac{1}{s}\left[e^{-s} - 1\right] = \frac{1 - e^{-s}}{s}$$

3)

a) $x(t) = u(t) - u(t-1) \overset{LT}{\longleftrightarrow} \frac{1}{s} - \frac{1}{s}e^{-s} = \frac{1}{s}\left[1 - e^{-s}\right]$

$u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s}$

$u(t-1) \overset{LT}{\longleftrightarrow} \frac{1}{s} \cdot e^{-s}$

OBS:

$x(t-t_0) \overset{LT}{\longleftrightarrow} X(s)\, e^{-st_0}$

b) $x(t) = 3e^{-3(t-2)} u(t-2)$

$e^{-3t} u(t) \overset{LT}{\longleftrightarrow} \frac{1}{(s+3)}$

OBS:

$e^{at} g(t) \overset{LT}{\longleftrightarrow} G(s-a)$

$3e^{-3(t-2)} u(t-2) \overset{LT}{\longleftrightarrow} 3 \cdot \frac{1}{(s+3)} e^{-2s} = \boxed{\frac{3e^{-2s}}{(s+3)}}$

c) $x(t) = 3e^{-3t} u(t-2) = 3e^{-3(t-2+2)} u(t-2) =$

$= 3 \cdot e^{-3(t-2)} \cdot e^{-6} u(t-2) = 3e^{-6} \cdot e^{-3(t-2)} u(t-2)$

$e^{-3(t-2)} u(t-2) \overset{LT}{\longleftrightarrow} \frac{e^{-2s}}{(s+3)}$

$x(t) \overset{LT}{\longleftrightarrow} 3e^{-6} \frac{e^{-2s}}{(s+3)} = \frac{3e^{-(2s+6)}}{(s+3)} = \frac{3e^{-2(s+3)}}{(s+3)}$

D) $x(t) = 5\,\text{sen}(\pi(t-1))\, u(t-1)$

$5\,\text{sen}(\pi t)\, u(t) \overset{LT}{\longleftrightarrow} \frac{5\pi}{s^2 + \pi^2}$

$x(t-t_0) \overset{LT}{\longleftrightarrow} X(s)\, e^{-st_0}$

$x(t) \overset{LT}{\longleftrightarrow} e^{-s} \cdot \frac{5\pi}{s^2+\pi^2}$

e) $x(t) = \delta(4t)$

$$\delta(t) \xleftrightarrow{LT} 1$$

$$x(t) \xleftrightarrow{LT} \frac{1}{4} \cdot 1 = \frac{1}{4}$$

f) $x(t) = u(4t)$

$$u(t) \xleftrightarrow{LT} \frac{1}{s}$$

$$u(4t) \xleftrightarrow{LT} \frac{1}{4} \frac{1}{\left(\frac{s}{4}\right)} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{s} = \boxed{\frac{1}{s}}$$

g) $x(t) = 5\,\text{sen}\,(2\pi(t-1))\, u(t-1)$

$$5\,\text{sen}\,(2\pi t)\, u(t) \xleftrightarrow{LT} \frac{10\pi}{s^2 + (2\pi)^2}$$

$$x(t) \xleftrightarrow{LT} e^{-s} \cdot \frac{10\pi}{s^2 + (2\pi)^2}$$

h) $x(t) = 5\,\text{sen}\,(2\pi t)\, u(t-1) = 5\,\text{sen}\,[2\pi(t-1+1)]\, u(t-1) =$

$$= 5\,\text{sen}\,[2\pi(t-1) + 2\pi]\, u(t-1)$$

$$\text{sen}\,[2\pi(t-1) + 2\pi] = \text{sen}\,[2\pi(t-1)]\, \overbrace{\cancel{\cos 2\pi}}^{1} + \overbrace{\cancel{\text{sen}\, 2\pi}}^{0} \cos[2\pi(t-1)]$$

$$x(t) = 5\,\text{sen}\,[2\pi(t-1)]\, u(t-1)$$

$$x(t) \xleftrightarrow{LT} e^{-s} \cdot \frac{10\pi}{s^2 + (2\pi)^2}$$

i) $x(t) = 2\cos(10\pi t)\cos(100\pi t)\,u(t)$

$$\cos a \cos b = \frac{1}{2}\left(\cos(a-b) + \cos(a+b)\right)$$

$$\cos(10\pi t)\cos(100\pi t) = \frac{1}{2}\left(\cos(-90\pi t) + \cos(110\pi t)\right) =$$

$$x(t) = \cos(110\pi t) + \cos(90\pi t)$$

$$x(t) \overset{LT}{\longleftrightarrow} \frac{s}{s^2+(110\pi)^2} + \frac{s}{s^2+(90\pi)^2}$$

$$=$$

j) $x(t) = 2e^{-5t}\cos(10\pi t)\,u(t)$ $\qquad e^{at}g(t) \overset{LT}{\longleftrightarrow} G(s-a)$

$$\cos(10\pi t)\,u(t) \overset{LT}{\longleftrightarrow} \frac{s}{s^2+(10\pi)^2}$$

$$x(t) \overset{LT}{\longleftrightarrow} \frac{2(s+5)}{(s+5)^2+(10\pi)^2}$$

k) $x(t) = 5\,\mathrm{sen}(\pi t - \pi/8)\,u(t)$

$$\mathrm{sen}(\pi t - \pi/8) = \mathrm{sen}\,\pi t \cos\pi/8 - \mathrm{sen}\,\pi/8 \cos\pi t$$

$$x(t) = 5\cos\left(\frac{\pi}{8}\right)\mathrm{sen}(\pi t)\,u(t) - 5\,\mathrm{sen}(\pi/8)\cos(\pi t)\,u(t)$$

$$x(t) \overset{LT}{\longleftrightarrow} 5\cos\left(\frac{\pi}{8}\right)\cdot\frac{\pi}{s^2+\pi^2} - 5\,\mathrm{sen}(\pi/8)\cdot\frac{s}{s^2+\pi^2}$$

l) $x(t) = \frac{d}{dt}\left(u(t-2)\right)$

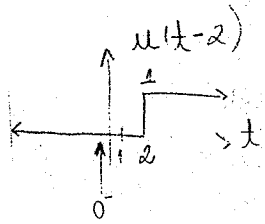
$u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s}$

$u(t-2) \overset{LT}{\longleftrightarrow} \frac{1}{s} e^{-2s}$

$x(t) \overset{LT}{\longleftrightarrow} \underline{\underline{e^{-2s}}}$

OBS:

$\frac{d}{dt}x(t) \overset{LT}{\longleftrightarrow} sX(s) - x(0^-)$



$\Rightarrow x(0^-) = 0$

m) $x(t) = \int_{0^+}^{t} u(\tau)\, d\tau \overset{LT}{\longleftrightarrow} \frac{1}{s^2}$

n) $x(t) = \frac{d}{dt}\left(5^{-(t-a)/2}\, u(t-a)\right)$

o) $x(t) = \frac{d}{dt}\left(e^{-10t} u(t)\right)$

$$u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s}$$

$$e^{-10t} u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s+10}$$

$$x(t) \overset{LT}{\longleftrightarrow} \frac{s}{s+10}$$

p) $x(t) = \frac{d}{dt}\left(4 \operatorname{sen}(10\pi t) u(t)\right)$

$$4 \operatorname{sen}(10\pi t) u(t) \overset{LT}{\longleftrightarrow} \frac{40\pi}{s^2 + (10\pi)^2}$$

$$x(t) \overset{LT}{\longleftrightarrow} \frac{40\pi s}{s^2 + (10\pi)^2}$$

q) $x(t) = \frac{d}{dt}\left(10 \cos(15\pi t) u(t)\right)$

$$10 \cos(15\pi t) u(t) \overset{LT}{\longleftrightarrow} \frac{10 s}{s^2 + (15\pi)^2}$$

$$x(t) \overset{LT}{\longleftrightarrow} \frac{10 s^2}{s^2 + (15\pi)^2}$$

1)

a) $X(s) = \dfrac{24}{s(s+8)} = \dfrac{A}{s} + \dfrac{B}{s+8} = \dfrac{3}{s} + \dfrac{(-3)}{s+8}$

ROC : Re {s} > 0

ROC : Re {s} > -8

$A = \dfrac{24}{0+8} = 3$

$x(t) = 3\,u(t) - 3\,e^{-8}u(t)$

$B = \dfrac{24}{-8} = -3$

b) $X(s) = \dfrac{20}{s^2+4s+3} = \dfrac{20}{(s+1)(s+3)} = \dfrac{A}{(s+1)} + \dfrac{B}{(s+3)} = \dfrac{10}{(s+1)} + \dfrac{(-10)}{s+3}$

ROC : Re {s} > -1

ROC : Re {s} > -3

$A = \dfrac{20}{(-1)+3} = \dfrac{20}{2} = 10$

$x(t) = 10\,e^{-t}u(t) - 10\,e^{-3t}u(t)$

$B = \dfrac{20}{(-3)+1} = \dfrac{20}{-2} = -10$

c) $X(s) = \dfrac{5}{s^2+6s+73} = \dfrac{5}{(s+3-j8)(s+3+j8)} = \dfrac{A}{(s+3-j8)} + \dfrac{B}{(s+3+j8)}$

$A = \dfrac{5}{(\cancel{-3}+j8)+\cancel{3}+j8} = \dfrac{5}{j16} = -j\,0{,}3125$

$B = \dfrac{5}{(\cancel{-3}-j8)+\cancel{3}-j8} = \dfrac{5}{-j16} = j\,0{,}3125$

ROC : Re{s} > -3 ROC : Re {s} > -3

$X(s) = \dfrac{-j0{,}3125}{(s+3-j8)} + \dfrac{j0{,}3125}{(s+3+j8)}$

$x(t) = -j0{,}3125\,e^{(-3+j8)t}u(t) + j0{,}3125\,e^{(-3-j8)t}u(t)$

$= -j0{,}3125\,e^{-3t}\cdot e^{j8t}u(t) + j0{,}3125\,e^{-3t}\cdot e^{-j8t}u(t)$

$= -j0{,}3125\,e^{-3t}\left[e^{j8t} - e^{-j8t}\right]u(t)$ $\Rightarrow$

$= \frac{(2j)}{2j} \cdot 0{,}3125\, e^{-3t} \left[\frac{e^{j8t} - e^{-j8t}}{2j} \right] u(t) =$

$= 0{,}625\, e^{-3t}\, \text{sen}\, 8t \quad u(t)$

$$X(s) = \frac{5}{s^2+6s+73} = \frac{5}{(s+3)^2+8^2} \cdot \frac{8}{8} = \frac{5}{8} \cdot \frac{8}{(s+3)^2+8^2}$$

$s^2+6s+73 =$

$s^2+6s+3^2+73-3^2$

$(s+3)^2+8^2$

$$x(t) = \frac{5}{8} \cdot e^{-3t}\, \text{sen}\, 8t \quad u(t)$$

D) $X(s) = \frac{10}{s(s^2+6s+73)} = \frac{10}{s(s+3-j8)(s+3+j8)} = \frac{A}{s} + \frac{B}{(s+3-j8)} + \frac{C}{(s+3+j8)}$

$$A = \frac{10}{(3-j8)(3+j8)} = \frac{10}{73}$$

$$B = \frac{10}{(-3+j8)(\cancel{-3}+j8+\cancel{3}+j8)} = \frac{10}{(-3+j8)(j16)} = \frac{10}{-128-j48} = -0{,}0685 + j0{,}0257$$

$$C = \frac{10}{(-3-j8)(\cancel{-3}-j8+\cancel{3}-j8)} = \frac{10}{(-3-j8)(-j16)} = \frac{10}{-128+j48} = -0{,}0685 - j0{,}0257$$

$$x(t) = \left[\frac{10}{73} + (-0{,}0685 + j0{,}0257)\, e^{(-3+j8)t} + (-0{,}0685 - j0{,}0257)\, e^{(-3-j8)t}\right]$$

$$x(t) = \left\{\frac{10}{73} + e^{-3t}\left[2(-0{,}0685)\frac{e^{j8t}+e^{-j8t}}{2} + 2j(j0{,}0257)(e^{j8t} - e^{-j8t})\right]\right\} u(t)$$

$$x(t) = \left\{\frac{10}{73} + e^{-3t}\left[-0{,}137 \cos 8t - 0{,}0514\, \text{sen}\, 8t\right]\right\} u(t)$$

e) $X(s) = \dfrac{4}{s^2(s^2+6s+73)} = \dfrac{4}{s^2(s+3-j8)(s+3+j8)} =$

$= \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{(s+3-j8)} + \dfrac{D}{(s+3+j8)}$

$A = \dfrac{4}{73}$

$B = \lim\limits_{s\to 0} \dfrac{d}{ds}\left[\dfrac{4}{s^2+6s+73}\right] = \lim\limits_{s\to 0}\left[-4\cdot\dfrac{(2s+6)}{(s^2+6s+73)^2}\right] = \dfrac{-24}{73^2} = \dfrac{-24}{5329}$

$C = \dfrac{4}{(-3+j8)^2(\cancel{-3}+j8+\cancel{3}+j8)} = 0{,}00225 + j\,0{,}00258$

$D = \dfrac{4}{(-3-j8)^2(-3-j8+3-j8)} = 0{,}00225 - j\,0{,}00258$

$x(t) = \left[\dfrac{4}{73}t - \dfrac{24}{5329} + (0{,}00225+j0{,}00258)e^{(-3+j8)t} + (0{,}00225-j0{,}00258)e^{(-3-j8)t}\right]u(t)$

$x(t) = \left\{\dfrac{4}{73}t - \dfrac{24}{5329} + e^{-3t}\left[2\cdot 0{,}00225\dfrac{(e^{j8t}+e^{-j8t})}{2} + 2j\cdot j0{,}00258\dfrac{(e^{j8t}-e^{-j8t})}{2j}\right]\right\}u(t)$

$x(t) = \left\{\dfrac{4}{73}t - \dfrac{24}{5329} + e^{-3t}\left[0{,}0045\cos 8t - 0{,}00516\,\text{sen}\,8t\right]\right\}u(t)$

f) $X(s) = \dfrac{2s}{s^2+2s+13} = \dfrac{2s}{(s+1-j3{,}46)(s+1+j3{,}46)} = \dfrac{A}{(s+1-j3{,}46)} + \dfrac{B}{(s+1+j3{,}46)}$

$$A = \frac{2(-1+j3{,}46)}{(-1+j3{,}46+1+j3{,}46)} = \frac{2(-1+j3{,}46)}{j6{,}92} = 1 + j0{,}289$$

$$B = \frac{2(-1-j3{,}46)}{(-1-j3{,}46+1-j3{,}46)} = \frac{2(-1-j3{,}46)}{-j6{,}92} = 1 - j0{,}289$$

$$x(t) = \left[(1+j0{,}289)\,e^{(-1+j3{,}46)t} + (1-j0{,}289)\,e^{(-1-j3{,}46)t}\right]u(t)$$

$$x(t) = \left\{ e^{-t}\left[\frac{\cdot 2\,(e^{j3{,}46t}+e^{-j3{,}46t})}{2} + \frac{\cdot 2j}{}\,j0{,}289\,(e^{j3{,}46t}-e^{-j3{,}46t})\right]\right\}u(t)$$

$$x(t) = \left\{ e^{-t}\left[2\cos 3{,}46t - 0{,}578\,\operatorname{sen} 3{,}46t\right]\right\}u(t)$$

g) $X(s) = \dfrac{s}{s+3} = 1 - \dfrac{3}{(s+3)} = \delta(t) - 3e^{-3t}u(t)$

$$\begin{array}{r|l} s & s+3 \\ \hline -s-3 & 1 \\ \hline -3 & \end{array}$$

h) $X(s) = \dfrac{s}{s^2-4s+4} = \dfrac{s}{(s-2)^2} = \dfrac{A}{(s-2)^2} + \dfrac{B}{(s-2)}$

$$A = \lim_{s\to +2} s = +2$$

$$B \lim_{s\to +2} \frac{d}{ds}\{s\} = 1$$

$$X(s) = \frac{+2}{(s-2)^2} + \frac{1}{(s-2)}$$

$$x(t) = +2te^{+2t}u(t) + e^{+2t}u(t)$$

i) $X(s) = \frac{s^2}{s^2-4s+4} = \frac{1}{} + \frac{(4s-4)}{s^2-4s+4} = 1 + \left\{ \frac{A}{(s-2)^2} + \frac{B}{(s-2)} \right\}$

$$\begin{array}{l|l} s^2 & s^2-4s+4 \\ \hline -s^2+4s-4 & 1 \\ \hline 4s-4 & \end{array}$$

$A = \lim_{s \to 2} \{4s-4\} = 4$

$B = \lim_{s \to 2} 4 = 4$

$X(s) = 1 + \left\{ \frac{4}{(s-2)^2} + \frac{4}{(s-2)} \right\}$

$x(t) = \delta(t) + 4te^{2t}u(t) + 4e^{2t}u(t)$

j) $X(s) = \frac{10s}{s^4+4s^2+4} = \frac{10s}{(s+j1,4)^2(s-j1,4)^2} =$

$= \frac{A}{(s+j1,4)^2} + \frac{B}{(s+j1,4)} + \frac{C}{(s-j1,4)^2} + \frac{D}{(s-j1,4)}$

$A = \lim_{s \to -j1,4} \left\{ \frac{10s}{(s-j1,4)^2} \right\} = \frac{10(-j1,4)}{(-j2,8)^2} = j1,79$

$B = \lim_{s \to -j1,4} \frac{d}{ds} \left\{ \frac{10s}{(s-j1,4)^2} \right\} = \lim_{s \to -j1,4} \left\{ \frac{10(s-j1,4)^2 - 10s \cdot 2(s-j1,4)}{(s-j1,4)^4} \right\} =$

$= \lim_{s \to -j1,4} \left\{ \frac{10(s-j1,4)^2 - 20s(s-j1,4)}{(s-j1,4)^4} \right\} = \frac{10(-j2,8)^2 - 20(-j1,4)(-j2,8)}{(-j2,8)^4}$

$B = 0$

$C = \lim_{s \to +j1,4} \left\{ \frac{10s}{(s+j1,4)^2} \right\} = \frac{10(j1,4)}{(j2,8)^2} = -j1,79$

$D = \lim_{s \to +j1,4} \left\{ \frac{10(s+j1,4)^2 - 10s \cdot 2(s+j1,4)}{(s+j1,4)^4} \right\} = \frac{10(j2,8)^2 - 20(j1,4)(j2,8)}{(j2,8)^4} = 0$

$$X(s) = \frac{j1{,}79}{(s+j1{,}4)^2} - \frac{j1{,}79}{(s-j1{,}4)^2}$$

$$x(t) = j1{,}79\, t\, e^{-j1{,}4t}\, u(t) - j1{,}79\, t\, e^{j1{,}4t}\, u(t)$$

$$x(t) = -j1{,}79\, t\, u(t) \left[\frac{e^{j1{,}4t} - e^{-j1{,}4t}}{2j} \right] \times 2j =$$

$$= 3{,}58\, t \operatorname{sen} 1{,}4t\; u(t)$$

3)

a) $\frac{d}{dt}\{x(t)\} + 10\,x(t) = u(t)$

$$s\,X(s) - x(0^-) + 10\,X(s) = \frac{1}{s}$$

$$X(s)(s+10) - 1 = \frac{1}{s}$$

$$(s+10)\,X(s) = \frac{s+1}{s}$$

$$X(s) = \frac{s+1}{s(s+10)} = \frac{A}{s} + \frac{B}{s+10}$$

$$A = \frac{1}{10} = 0{,}1$$

$$B = \frac{-9}{-10} = \frac{9}{10} = 0{,}9$$

$$x(t) = 0{,}1\,u(t) + 0{,}9\,e^{-10t}\,u(t)$$

b) $\frac{d^2}{dt^2}\{x(t)\} - 2\frac{d}{dt}\{x(t)\} + 4x(t) = 0$

$$s^2 X(s) - s\,\cancel{x(0^-)}^{\,0} - \cancel{x'(0^-)}^{\,4} - 2\{s X(s) - \cancel{x(0^-)}^{\,0}\} + 4X(s) = 0$$

$$s^2 X(s) - 4 - 2s X(s) + 4X(s) = 0$$

$$X(s)(s^2 - 2s + 4) = 4$$

$$X(s) = \frac{4}{s^2 - 2s + 4} = \frac{4}{(s-1-j1{,}73)(s-1+j1{,}73)} =$$

$$= \frac{A}{(s-1-j1{,}73)} + \frac{B}{(s-1+j1{,}73)}$$

$$A = \frac{4}{(1+j1{,}73-1+j1{,}73)} = \frac{4}{j3{,}46} = -j1{,}156$$

$$B = j1{,}156$$

$$x(t) = -j1{,}156\,e^{(1+j1{,}73)t}u(t) + j1{,}156\,e^{(1-j1{,}73)t}u(t)$$

$$= e^{t}\left(-j1{,}156 \times 2j \cdot \frac{(e^{j1{,}73t} - e^{-j1{,}73t})}{2j}\right)u(t)$$

$$= 2{,}312\,e^{t}\,\text{sen}\,1{,}73t\;u(t)$$

c) $\frac{d}{dt}\{x(t)\} + 2x(t) = \operatorname{sen}(2\pi t)\, u(t)$

$$s\,\cancel{X(s)} - x(0^{-}) + 2X(s) = \frac{2\pi}{s^2+(2\pi)^2} \qquad x(0^-) \to -4$$

$$X(s)\,(s+2) - 4 = \frac{2\pi}{s^2+(2\pi)^2}$$

$$X(s) = \frac{4\,(s^2+(2\pi)^2)+2\pi}{(s+2)\,(s^2+(2\pi)^2)}$$

$$X(s) = \frac{4\,(s^2+(2\pi)^2)+2\pi}{(s+2)\,(s-j2\pi)\,(s+j2\pi)} = \qquad s^2 = -(2\pi)^2 \qquad s = \pm j2\pi$$

$$= \frac{A}{(s+2)} + \frac{B}{(s-j2\pi)} + \frac{C}{(s+j2\pi)}$$

$$A = \frac{4\,((-2)^2+(2\pi)^2)+2\pi}{(-2-j2\pi)\,(-2+j2\pi)} = 4{,}14$$

$$(-2-j2\pi)(-2+j2\pi)$$
$$4 - 2j2\pi + \cancel{2j2\pi} - j^2(2\pi)^2$$
$$4 + (2\pi)^2$$

$$B = \frac{4\,((j2\pi)^2+(2\pi)^2)+2\pi}{(2+j2\pi)\,(j4\pi)} = -0{,}0723 - j0{,}023$$

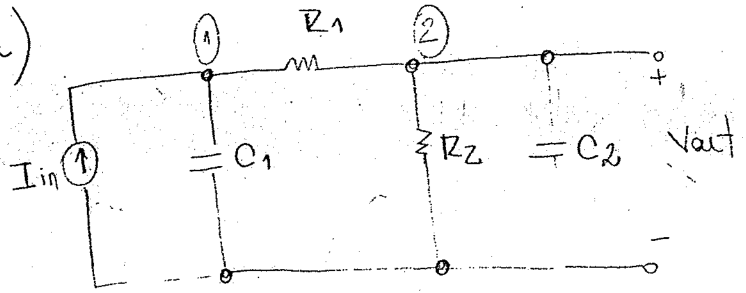
$$C = -0{,}0723 + j0{,}023$$

$$x(t) = 4{,}14\,e^{-2t}u(t) - 0{,}1446\cos(2\pi t)\,u(t) + 0{,}046\operatorname{sen}(2\pi t)\,u(t)$$

$$x(t) = 4{,}14\,e^{-2t}u(t) + (-0{,}0723 - j0{,}023)\,e^{j2\pi t}u(t) + (-0{,}0723 + j0{,}023)\,e^{-j2\pi t}u(t) =$$

$$= 4{,}14\,e^{-2t}u(t) - 2\cdot 0{,}0723\left[\frac{e^{j2\pi t}+e^{-j2\pi t}}{2}\right]u(t) - 2j\cdot j0{,}023\left[\frac{e^{j2\pi t}-e^{-j2\pi t}}{2j}\right]u(t)$$

a)



Nó 2:

$$\frac{V_{out} - V_1}{R_1} + \frac{V_{out}}{R_2} + \frac{V_{out}}{\frac{1}{sC_2}} = 0$$

$$\frac{V_{out}}{R_1} - \frac{V_1}{R_1} + \frac{V_{out}}{R_2} + sC_2 V_{out} = 0$$

$$V_{out}\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_2\right) = \frac{V_1}{R_1}$$

$$V_1 = V_{out}\left(1 + \frac{R_1}{R_2} + sR_1C_2\right) = V_{out}\left(1 + 2 + 0{,}002s\right) = (0{,}002s + 3)V_{out}$$

Nó 1:

$$-I_{in} + \frac{V_1}{\frac{1}{sC_1}} + \frac{V_1 - V_{out}}{R_1} = 0$$

$$sC_1 V_1 + \frac{V_1}{R_1} - \frac{V_{out}}{R_1} = I_{in}$$

$$V_1\left(sC_1 + \frac{1}{R_1}\right) - \frac{V_{out}}{R_1} = I_{in}$$

$$(0{,}002s + 3)V_{out}\left(sC_1 + \frac{1}{R_1}\right) - \frac{V_{out}}{R_1} = I_{in}$$

$$V_{out}\left[(0{,}002s + 3)\left(sC_1 + \frac{1}{R_1}\right) - \frac{1}{R_1}\right] = I_{in}$$

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{1}{6 \cdot 10^{-9}s^2 + 1 \cdot 10^{-5}s + 1 \cdot 10^{-3}} = \frac{1}{6 \cdot 10^{-9}(s^2 + 1666\,67 + 166666{,}67)} =$$

$$Y(s) = \frac{1}{6 \cdot 10^{-9}s^3 + 10^{-5}s^2 + 10^{-3}s} = \frac{\frac{1}{6 \cdot 10^{-9}}}{s\,(s + 106{,}85)(s + 1559{,}82)}$$

$$= \frac{A}{s} + \frac{B}{(s+106,85)} + \frac{C}{(s+1559,82)}$$

$$A = \frac{\frac{1}{6 \cdot 10^{-9}}}{(106,85)(1559,82)} = 1000$$

$$B = \frac{1/6 \cdot 10^{-9}}{(-106,85)(-106,85+1559,82)} =$$

$$C = \frac{1}{(-1559,82)(-1559,82+106,85)} =$$

$$y(t) = 6 \cdot 10^{-6}\,u(t) - \quad 10^{-6}\, e^{-106,85t}\,u(t) + \quad e^{-1559,82t}\,u(t)$$

b)

a) $x(t) = \text{sen}(2\pi t)\, u(t)$

$$H(s) = \frac{1}{s+1}$$

$$Y(s) = \frac{1}{s+1} \cdot \frac{2\pi}{s^2+(2\pi)^2} = \frac{2\pi}{(s+1)(s+j2\pi)(s-j2\pi)} = \frac{A}{s+1} + \frac{B}{(s+j2\pi)} + \frac{C}{(s-j2\pi)}$$

$$A = \frac{2\pi}{(-1+j2\pi)(-1-j2\pi)} = \frac{2\pi}{1+4\pi^2} = 0{,}155$$

$$B = \frac{2\pi}{(-j2\pi+1)(-j4\pi)} = -7{,}76 \cdot 10^{-2} + j\,1{,}235 \cdot 10^{-2}$$

$$C = -7{,}76 \cdot 10^{-2} - j\,1{,}235 \cdot 10^{-2}$$

$$y(t) = 0{,}155\, e^{-t} u(t) + (-7{,}76 \cdot 10^{-2} + j\,1{,}235 \cdot 10^{-2})\, e^{-j2\pi t} u(t) + (-7{,}76 \cdot 10^{-2} - j\,1{,}235 \cdot 10^{-2})\, e^{j2\pi t} u(t)$$

$$y(t) = 0{,}155\, e^{-t} u(t) - 7{,}76 \cdot 10^{-2} \left[e^{j2\pi t} + e^{-j2\pi t}\right] u(t) - j\,1{,}235 \cdot 10^{-2} \left[\frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}\right] u(t) \times 2j$$

$$y(t) = 0{,}155\, e^{-t} u(t) - 0{,}155 \cos 2\pi t\, u(t) + 0{,}0247\, \text{sen}\, 2\pi t\, u(t)$$

$$H(j2\pi) = \frac{1}{1+j2\pi} = 0{,}0247 - j\,0{,}155$$

$$Y(s) = \frac{A}{s+1} + \text{Re}\{H(j2\pi)\} \frac{2\pi}{s^2+(2\pi)^2} + \text{Im}\{H(j2\pi)\} \frac{s}{s^2+(2\pi)^2}$$

$$y(s) = 0{,}155\, e^{-t} u(t) + 0{,}0247\, \text{sen}\, 2\pi t\, u(t) - 0{,}155 \cos 2\pi t\, u(t)$$

$\Rightarrow x(t) = \text{sen}(2\pi t)\,u(t)$

$$H(S) = \frac{5S}{S^2+2S+2}$$

$$H(j2\pi) = \frac{j10\pi}{(j2\pi)^2 + j4\pi + 2} = 0{,}253 - j0{,}754$$

b) $x(t) = u(t)$

$$H(s) = \frac{3}{s+2}$$

$$Y(s) = \frac{3}{s(s+2)} = \frac{A}{s} + \frac{B}{(s+2)} \Rightarrow y(t) = \frac{3}{2}u(t) - \frac{3}{2}e^{-2t}u(t)$$

$$A = \frac{3}{2}$$

$$B = -\frac{3}{2}$$

c) $$Y(s) = \frac{3\cancel{s}}{\cancel{s}(s+2)} = \frac{3}{(s+2)} \rightarrow y(t) = 3e^{-2t}u(t)$$ //

D) $$Y(s) = \frac{5}{s^2+2s+2} = \frac{5}{(s+1-j)(s+1+j)} = \frac{A}{(s+1-j)} + \frac{B}{(s+1+j)}$$

$$A = \frac{5}{(\cancel{-1+j}+1+j)} = \frac{5}{2j} = -j2{,}5$$

$$B = j2{,}5$$

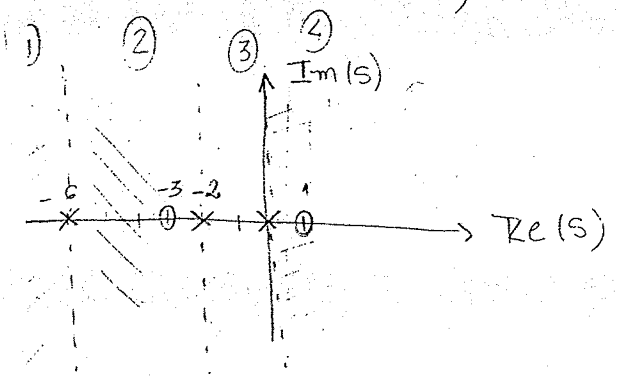
$$y(t) = -j2{,}5e^{(-1+j)t}u(t) + j2{,}5e^{(-1-j)t}u(t)$$

$$y(t) = e^{-t}\left[-j2{,}5 \cdot 2j\frac{(e^{jt} - e^{-jt})}{2j}\right]u(t) =$$

$$= 5e^{-t}\,\text{sen}\,t\,u(t)$$

$$\frac{5}{s^2+2s+2} = \frac{5}{(s+1)^2+1^2} \xrightarrow{\mathcal{L}^{-1}} 5e^{-t}\,\text{sen}\,t\,u(t)$$

a) $H(s) = \dfrac{(s+3)(s-1)}{s(s+2)(s+6)} = \dfrac{A}{s} + \dfrac{B}{(s+2)} + \dfrac{C}{s+6}$



$A = \dfrac{(3)(-1)}{(2)(6)} = -\dfrac{1}{4}$

$B = \dfrac{(-2+3)(-2-1)}{-2\,(-2+6)} = \dfrac{1\cdot -3}{-2\cdot(4)} = \dfrac{3}{8}$

$C = \dfrac{(-6+3)(-6-1)}{-6\,(-6+2)} = \dfrac{-3\cdot -7}{-6\cdot(-4)} = \dfrac{21}{24} = \dfrac{7}{8}$

2

$-6 < Re\{s\} < -2$

$\rightarrow$ ROC : $Re\{s\} < 0$

$H(s) = \dfrac{-\frac{1}{4}}{s} + \dfrac{\frac{3}{8}}{(s+2)} + \dfrac{\frac{7}{8}}{(s+6)}$ $\rightarrow$ ROC : $Re\{s\} > -6$

$\rightarrow$ ROC: $Re\{s\} < -2$

$y(t) = H^{-1}(s) = \frac{1}{4} e^{-0t} u(-t) - \frac{3}{8} e^{-2t} u(-t) + \frac{7}{8} e^{-6t} u(t)$

$y(t) = \frac{1}{4} u(-t) - \frac{3}{8} e^{-2t} u(-t) + \frac{7}{8} e^{-6t} u(t)$

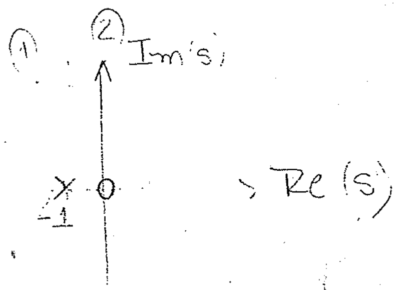
b) $H(S) = \dfrac{S}{S^2+S+1} = \dfrac{S}{(S+0{,}5-j0{,}866)(S+0{,}5+j0{,}866)} =$

$= \dfrac{A}{(S+0{,}5-j0{,}866)} + \dfrac{B}{(S+0{,}5+j0{,}866)}$

$A =$

c) $H(s) = \dfrac{s\,\cancel{(s+1)}}{(s+1)(s+1)} = \dfrac{s}{s+1} = 1 - \dfrac{1}{s+1}$

$$\begin{array}{r|l} s & s+1 \\ \hline -s-1 & 1 \\ \hline -1 & \end{array}$$



② $Re(s) > -1$

$y(t) = \delta(t) - e^{-t}\,u(t)$

① $Re(s) < -1$

$y(t) = \delta(t) + e^{-t}\,u(-t)$